Mismatch, Rematch, and Investment

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Preliminary and Incomplete. This Version: May 2009

Abstract

Mobility depends essentially on investment, which often occurs in environments in which individuals match (school) or will match after investing (the labor market). Where partners can transfer surplus to each other only imperfectly (NTU), the pattern of matching will typically be inefficient, involving too much segregation, and provide a possible rationale for "associational redistribution" such as affirmative action: a social planner who could enforce a matching outcome that differs from the market outcome may raise aggregate social surplus. We show that this static inefficiency due to NTU can be exacerbated in a dynamic environment, in which investments made before the match determine individuals' productive types. In contrast to TU models there will typically be investment distortions, with high types over-investing and low types under-investing, which amplifies inequality. We study several forms of associational redistribution, assessing the differential effects of achievement-based and background-based polices; early-stage and later-stage policies; and their interaction.

Keywords: Matching, nontransferable utility, affirmative action, segregation, education.

JEL: C78, I28, H52, J78.

1 Introduction

Some of the most important economic decisions we make - where to live, which profession to enter, whom to marry - depend for their consequences not only

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on our own characteristics or "types" (wealth, skill, or temperament), but also on those of the people with whom we live or work. These decisions matter not only statically, for our own well-being or those of our partners, but also dynamically: the prospect of being able to select particular kinds of neighbors, associates or mates, or the environment those partners provide, affect the costs and benefits of investment. The impact of those investments may extend far beyond our immediate partners to the economy as a whole.

The natural question – one in which policy makers in rich and poor countries have taken a direct interest – is whether the market outcome of our "matching" decisions leads to outcomes that are socially desirable. In fact, it has been often contended in public policy debates in the U.S., U.K., India and elsewhere that the market often *has* failed to sort people desirably: there is too much segregation (by educational attainment, ethnic background or caste); certain groups appear to be "excluded" from normal participation in economic life; and that in turn depresses their willingness to invest in human capital. If this assessment of possible market mis-match is correct, policy remedies may include re-matching individuals into other partnerships via affirmative action or school integration.

Economic theory makes it clear that some form of "imperfection" needs to be present in order for such policy intervention to be justified. If the characteristics of matched partners (ability, gender, or race) are exogenous, then under the assumptions that (1) partners can make non-distortionary side payments to each other (transferable utility or TU); (2) there is symmetric information about characteristics; and (3) there are no widespread externalities, stable matching outcomes are social surplus maximizing: no other assignment of individuals can raise the economy's aggregate payoff. Neither are outcomes likely to be worthy of policy intervention when characteristics (such as income or skill) are endogenous, the result at least in part of investments made either before matching or within matches (Cole et al., 2001, Felli and Roberts, 2002).

Absent imperfections, then, the above concerns about the nature of actual market outcomes would appear to be unjustified. But there are, of course, many reasons to suspect that imperfections may be pertinent: search frictions, widespread externalities, and statistical discrimination have all been studied as possible sources of matching market failure that can generate inefficient levels of output and investment, or undesirable degrees of inequality. The latter in particular has been cited as a justification for policy intervention that directly interferes with the sorting outcome through re-matching, that is *associational redistribution* (AR) (Durlauf, 1996a). Examples include affirmative action (see Fryer and Loury, 2007), school integration, or certain types of labor subsidies that target the less qualified. AR has also been supported on efficiency grounds, in the case where there is a problem of statistical discrimination: Coate and Loury (1993) provides one formalization of the argument that equilibria where under-investment is supported by "wrong" expectations may be eliminated by affirmative action policies (an "encouragement effect"), but importantly also points out a possible downside ("stigma effect").

This paper emphasizes another source of inefficient stable matching: non-transferable utility (NTU) within matches. In many situations circumstances place bounds on compensations to or from people we interact with, for instance through the legal framework, because of capital market imperfections and moral hazard within firms, or out of "behavioral" considerations. Lack of or imperfect access to credit markets, limited liability and moral hazard within a firm generally prevent non-distortionary side payments; deviations from the payoffs supporting the second best allocation decrease joint surplus. This applies to law firms and other partnerships that use profit sharing arrangements, but also to industry firms operating some form of efficiency wage schedule. Moreover, when part of the compensation is inalienable, such as training or reputation, transferring individual gains may become very costly.

It is known at least since Becker (1973) that under nontransferable utility the equilibrium matching pattern need not maximize aggregate social surplus (see also Legros and Newman, 2007). This is because the types that may have received large shares of the pie generated in an (efficient) match under TU, will now receive a smaller share to due to rigidities in dividing that pie, and so they will prefer to match with types with whom they can obtain large payoffs. In some cases this will lead to what appears as excessive segregation.

It is useful to distinguish among three distinct but interacting distortions that occur in NTU matching models. We refer to them as inefficiency *of* the match, *by* the match, and *for* the match. *By-the-match* inefficiency results when the Pareto frontier for matched agents does not coincide with an iso-surplus surface; matched partners need not maximize their own joint surplus, and aggregate performance is sensitive to the distribution of surplus within matches (see e.g. Legros and Newman, 2008). As we have suggested, it may cause *Of-the-match* inefficiency, which refers to the kind of mis-match pointed out by Becker, wherein reassignments of partners may raise the aggregate welfare. *For-the-match* inefficiency results from the first two: since surplus shares and levels are distorted in a laissez-faire match, so are incentives to invest before it happens. In particular, NTU inhibits the market's ability to attract an efficient distribution of skills by signaling scarcity through adjustments in wages or other surplus shares. Returns to skills in short supply may not rise enough to encourage investment, while returns to plentiful skills may remain too high, encouraging overinvestment.

Thus NTU provides an efficiency-based rationale for AR policy, at least if one accepts an "ex-ante" Pareto optimality criterion, i.e., maximizing welfare from behind a veil of ignorance, before people know their types (as in Harsanyi, 1953, Holmström and Myerson, 1983). Though it cannot directly address the sources of NTU and therefore by-the-match inefficiency, it does provide an instrument for correcting inefficiency of the match. A possible pitfall of such policies – one that has often been made in policy debates – is that they may harm the investment incentives of the group favored by the policy by guaranteeing its members minimal payoffs. Moreover, they may reduce the incentives of *unfavored* groups, whose members may get high returns under laissez faire. Against this is the possibility that a properly designed AR policy results in payoffs that more closely replicate the TU outcome, so that efficient investment incentives might be partially restored. In any case, an overall assessment of AR policy must therefore take account of their effects on investment incentives, i.e., on whether they mollify or exacerbate for-the-match inefficiency.[1]

In general, the interaction of all three distortions must be taken into account for assessing policy; here, we shall shut down inefficiency by the match in order to focus on the other two distortions by assuming *strict* NTU, i.e. the Pareto frontier is a single point. This also allows us to focus on redistributive policies that are purely associational, since transfers such as taxes and subsidies would be hard to implement in this environment.[2]

The setup we employ to analyze various forms of associational redistribution is as follows. Agents have a binary background type reflecting whether they

[1]And quite apart from whether such policy is desirable, it is of interest to predict its likely effects; for instance, variations in aggregate surplus in the model may correspond to variations in national income across countries following different policies.

[2]However, they may be approximated to varying degrees by allowing for randomized matching policies; here we will focus on deterministic policies and defer consideration of the broader menu of policies to another paper.

are *privileged* or not. Privilege confers a productivity benefit, either in terms of (increased) labor market output or (reduced) education cost. Agents can affect their labor-market productivity (also a binary variable) by investing in education, which determines the probability of becoming a high-achievement type. In the labor market agents match into firms whose output depends on members' achievement and possibly background. The production technology is such that diversity (heterogeneity) within firms is more productive, and would be the outcome under unrestricted side payments. We model nontransferable utility in the simplest possible way: output is shared equally within firms.

As a result the labor market segregates in educational achievement and background. Thus, the laissez faire equilibrium outcome is inefficient from an aggregate surplus perspective, and there is likely to be *overinvestment at the top* and *underinvestment at the bottom*: the underprivileged find investing to be too costly or unremunerative, while the privileged receive inefficiently high rewards in the labor market.

We then evaluate several associational redistribution policies. When labor market segregation in education is inefficient, an immediate remedy is an *achievement based* policy rematching agents based on educational attainment. For instance, "Workfare" and certain European labor market policies provide wage subsidies for employing long-term unemployed or low educated youth. But in a dynamic setting with investments, a trade-off emerges. Though rematching increases output, investment incentives are depressed: the policy raises the returns to low education outcomes and lowers the return to high outcomes. The adverse incentive effect may be partially mitigated by a rematching policy that conditions not on results of choices but on exogenous information correlated with education outcomes, such as agents' socioeconomic status. Examples of such *background based* policy are race- or gender-based affirmative action.

When agents' types enter the production function directly, returns from education investment depend positively on the quality of the match an agent obtains on the labor market. Then a background based policy may serve to encourage underprivileged agents to invest and mitigate both underinvestment at the bottom and overinvestment at the top.

Matching may be pertinent at the investment stage as well as in the labor market, since investments are often taken not in solitude but in schools or neighborhoods where peer effects matter. Thus associational redistribution both at early and late stages might be justified, and optimal timing and coordination of such interventions becomes a concern. A policy of *school integration* reduces segregation of schools with respect to background. While this policy serves to extend access to education, it does not further interfere with a laissez-faire labor market allocation. Hence, school integration is beneficial if it is cost efficient at the schooling stage, and often dominates the labor market interventions.

One can also ask how policies used in the labor market *and* the investment stage interact, and ask whether school integration is a substitute or a complement to labor market rematching policy. In addition, we can consider a policy that is dynamic in the sense that the rematch in the labor market conditions on agents choices of investment environments. Such a *club based* policy is sometimes used in regulating university access by assigning quotas to high-schools or neighborhoods (for instance the Texas 10 percent law).

The literature on school and neighborhood choice (see among others Bénabou, 1993, 1996, Epple and Romano, 1998) typically finds too much segregation in types. This may be due to market power (see e.g. Board, 2008) or widespread externalities (see also Durlauf, 1996b, Fernández and Rogerson, 2001). When attributes are fixed, aggregate surplus may be raised by an adequate policy of bribing some individuals to migrate (see also de Bartolome, 1990). Fernández and Galí (1999) compare matching market allocations of school choice with those generated by tournaments: the latter may dominate in terms of aggregate surplus when capital market imperfections lead to nontransferabilities. They do not consider investments before the match.

Peters and Siow (2002) and Booth and Coles (2009) present models where agents invest in attributes before matching on a marriage market under strictly nontransferable utility. Investments are taken in solitude, so peer effects are absent. The former finds that allocations are constrained Pareto optimal (with the production technology they study, aggregate surplus is also maximized), and does not discuss policy. The latter compares different marriage institutions in terms of their impact on matching and investments. Gall et al. (2006) analyze the impact of timing of investment on allocative efficiency. Several recent studies consider investments before matching under asymmetric information (see e.g. Bidner, 2008, Hopkins, 2005, Hoppe et al., 2008), mainly focusing on wasteful signaling, while not considering associational redistribution.

Finally, the emphasis here is on characterizing stable matches and contrasting with ones imposed by policy. Thus we shall not be concerned here with the market outcome under search frictions (Shimer and Smith, 2000, Smith, 2006), nor on mechanisms that might be employed to achieve either stable matches or ones with desirable welfare properties (e.g. Roth and Sotomayor, 1990). Stable matches in this paper may, of course, be attained using matching mechanisms.

The paper proceeds as follows. Section 2 lays out the labor market and discusses effectiveness of policies of associational redistribution in terms of sorting, incentives, and exclusion. Section 3 presents the schooling stage and a policy of school integration. Section 4 considers effectiveness of policies at both schooling stage and labor market and introduces club based policies. Section 5 concludes, and the appendix contains the more tedious calculations.

2 Labor Market

The market is populated by a continuum of agents I with unit measure. Though we refer to it as a "labor market," it can also be interpreted in other ways, for instance as a market for places in university. Agents are characterized by their educational attainment a which is either high h or low ℓ (in the university interpretation, these would be secondary school achievements). Denote the measure of h agents by $q \in [0, 1]$. In the market, agents match into firms of size two and jointly produce output. Profit y in a firm depends on agents' education outcomes. Assume that profits increase in attainment,

$$y(\ell, \ell) < y(\ell, h) = y(h, \ell) < y(h, h).$$

Profits in firms have the *desirability of diversity* property if

$$2y(h, \ell) > y(h, h) + y(\ell, \ell). \tag{DD}$$

DD holds for instance when a is real-valued and y a concave function of the sum of the types. It could also be the result of a technology that combines two different tasks, one human capital intensive and one less so, say engineering and design versus actual manufacturing, with the firm free to assign the worker to the task (Kremer and Maskin, 1996). Denote by $w(a, a')$ the wage of an agent with educational attainment a when matching with an agent whose attainment is a'. Wages are positive and sum up to the firm's profit, $w(a, a') + w(a', a) = y(a, a')$. Agents are risk neutral and derive utility from wage income.

To provide a benchmark solve now for the competitive labor market equilibrium, that is a stable match of agents into firms. With DD there are wages

$w(h,\ell) \geq 0$ and $w(\ell,h) \geq 0$ with $w(h,\ell) + w(h,\ell) = y(h,\ell)$ such that

$$w(h,\ell) \geq y(h,h)/2 \text{ and } w(\ell,h) > w(\ell,\ell)/2.$$

This implies that given wages $w(.)$ there is no distribution of profits in segregated firms such that agents in integrated firms were better off forming a segregated firm. Hence, in labor market equilibrium measure $\min\{q, 1-q\}$ of integrated firms emerge, the remainder segregates. Market wages are determined by scarcity, that is $w(h,\ell) = y(h,h)/2$ if $q > 1/2$, $w(\ell,h) = y(\ell,\ell)/2$ if $q < 1/2$, and $w(h,\ell) \in [y(h,h)/2, y(h,\ell) - y(\ell,\ell)/2]$ if $q = 1/2$.

2.1 Nontransferable Utility

The example above tacitly assumed that utility was perfectly transferable on the labor market. This means agents can contract on the distribution of profit in a firm without affecting productive efficiency, i.e. firm profit. For a number of plausible reasons this assumption may often be violated. Lack of access to or imperfections on the credit market, limited liability and moral hazard within the firm are one reason not to expect perfectly transferable utility. Others are incomplete contracts and renegotiation, risk aversion, legal constraints and regulation, or behavioral concerns.

To facilitate exposition we assume an extreme case of non-transferabilities, namely strictly nontransferable utility, so that only a single vector of payoffs to agents is feasible in any firm.[3] To minimize notation, assume that profits are shared equally in firms, that is

$$y(h,h) = 2W, y(h,\ell) = y(\ell,h) = 2w, y(\ell,\ell) = 0.$$

Assume as a normalization that $W < 1$. Then wages are typically bounded above by 1, which permits interpretation of investments induced by expected wages, introduced below, as probabilities.

Nontransferable utility affects the equilibrium labor market allocation quite substantially. Despite $2w > W$, i.e. DD holds, integration is no longer possible in equilibrium. Suppose that a positive measure of (h,ℓ) firms form and h agents obtain wage w. Then any two h agents have a profitable deviation by

[3] All results in the paper are robust to allowing for some transferability by admitting for either a sufficiently small amount of perfect transferability, or for sufficient curvature in the Pareto frontier within teams, see Appendix.

starting a (h, h) firm earning W each, a contradiction to stability. Hence, under strictly nontransferable utility only homogeneous firms emerge.

If both high and low types have positive measure ($0 < q < 1$), and diversity is desirable in production ($2w > W$), aggregate surplus is strictly lower when utility is nontransferable. If $q \leq 1/2$, surplus is $2qw$ if utility is transferable, while it is qW if not; if $q > 1/2$, surplus is $(1-q)2w + (2q-1)W$ if utility is transferable, and qW if not.

Non-transferability of utility may therefore distort the matching pattern and reduce aggregate surplus. This seems to provide a first-order rationale for associational redistribution on the labor market. Moreover, studying such interventions may yield insights about the output effects of policies.

Consider a policy of associational redistribution that assigns h agents to ℓ agents whenever possible. Any remaining agents are rationed uniformly into homogeneous firms. Call this an *achievement based* policy. This policy replicates the matching pattern under transferable utility and achieves an increase in aggregate surplus for any exogenously given distribution of educational attainments, as measured by q in our example. However, the non-favored group of type h is clearly worse off under the policy.

Active labor market policy often resembles achievement based policies, e.g. employment subsidies. By targeting the long term or young unemployed such policy effectively rematches the labor market conditional on educational achievements or rather lack thereof. In many industrialized countries some variety of wage subsidy or workfare program was used: the Targeted Jobs Tax Credit (TJTC) and later on the Work Opportunity Tax Credit (WOTC) in the US, as part of the Hartz policy reform in Germany, in the New Deal for Young People in the UK, and payroll tax subsidies for minimum wage labor contracts and wage subsidies for the unemployed young in France.

2.2 Education Investments

An often voiced critique of associational redistribution is that it spoils incentives. Educational attainments, and agents' attributes on markets in general, often result from individual choices, which are affected by the anticipated rewards accruing to the various types. Therefore endogenizing types is a natural way to assess such critique.

Suppose then that an exogenously given measure $\pi \in [0, 1]$ of agents can invest in education. π is best understood as the fraction of the population with access to schooling. When investing, agents exert effort $e \in [0, 1]$ to acquire education. Effort e, which is never verifiable, comes at a utility cost $e^2/2$. Spending effort e yields a high education outcome h with probability e and a low education outcome ℓ otherwise. The measure of high achievers (agents with attainment h), q, is now endogenous and given by $q = \pi e$.

Events unfold as follows.

- Measure π of agents simultaneously choose e, the remaining agents are assigned $e = 0$.
- Educational outcomes are realized and publicly observed.
- Agents form a stable match (in case of laissez faire) or are matched in accordance with whichever policy is in effect.

Let $w(h)$ and $w(\ell)$ be the rationally anticipated wages of a high and a low achiever. Then, an investing agent solves

$$\max_e e[w(h) - w(\ell)] + w(\ell) - \frac{1}{2}e^2,$$

yielding $e = w(h) - w(\ell)$.

In the benchmark case, when utility is fully transferable, investment incentives depend on whether q is anticipated to be greater or lower than $1/2$. If $q > 1/2$, high achievers are in excess supply and have a chance of being matched with a low or a high achiever. As a pair of high achievers obtain $w(h) = W$ each, this is also what they get when matched with a low achiever,[4] who gets the residual $w(\ell) = 2w - W$. Hence, investments are $e = 2(W - w)$. If $q < 1/2$, low achievers are in excess supply, obtain $w(\ell) = 0$, and high achiever match only with low achievers obtaining $w(h) = 2w$. Investments are $e = \min\{2w; 1\}$, strictly greater than in the case $q > 1/2$. In equilibrium the anticipated q coincides with its realization πe, for instance, if $q > 1/2$, $e^{TU} = 2(W - w)$ and therefore we need that $\pi 2(W - w) > 1/2$. We have the following result (all proofs missing in the text are in the appendix).

Lemma 1 *Let π be the measure of agents who are able to invest. Suppose that utility is fully transferable.*

[4]Equal treatment on the labor market holds under transferability: if one high achiever gets strictly more than another one, the latter can match with the partner of the former at a wage that is slightly lower.

(i) If $W - w > \frac{1}{4\pi}$*,* $e^{TU} = 2(W - w)$ *and* $q > 1/2$*,*

(ii) If $W - w < \frac{1}{4\pi} < \min\{w; 1/2\}$*,* $e^{TU} = \frac{1}{2\pi}$ *and* $q = 1/2$*,*

(iii) If $\min\{w; 1/2\} < \frac{1}{4\pi}$*,* $e^{TU} = \min\{2w; 1\}$ *and* $q < 1/2$*.*

When utility is strictly nontransferable, the labor market segregates as derived above. Therefore $w(h) = W$, $w(\ell) = 0$, and the laissez-faire investment is $e^{LF} = W$. By Lemma 1 the social return from education, and thus e^{TU}, decreases with q. Under nontransferable utility the private return from education is independent of q. Hence, given W, the difference in investment under nontransferable and transferable utility increases in q. Therefore a corollary follows from the lemma.

Corollary 1 *Comparing investment levels when utility is perfectly transferable (TU) and strictly nontransferable (LF) yields*

$$e^{LF} > e^{TU} \Leftrightarrow W > \frac{1}{2\pi}.$$

Thus, both *overinvestment* relative to the benchmark (if $\pi W > 1/2$), since $W > 2(W - w)$ and *underinvestment* (if $\pi W < 1/2$) are possible. Those who have no access, of course, always underinvest relative to what they would do had they access.

2.3 Achievement Based Policy

Since mismatches due to nontransferable utility distort investment incentives, the case for associational redistribution seems even more compelling when the measure of high achievers is endogenous. This intuition is incomplete, however, because *given nontransferabilities on the labor market* enforcing the "correct" sorting may in fact worsen investment incentives substantially.

Recall that under transferable utility the labor market wage adjusts as to provide the long market side with its autarky payoff (i.e. W for h and 0 for ℓ agents). For instance, if low achievers are in excess supply they obtain 0 and high achievers get $2w$. If utility is strictly nontransferable, however, low achievers have strictly positive payoff under an achievement based policy, since they get w with a positive probability due to uniform rationing. High achievers get w for sure. Hence, investment incentives are weaker than under laissez-faire as the return is lower in the high achievement state h and higher in the low achievement state ℓ. Indeed, it is not hard to show that in any equilibrium under forced integration low achievers must be in excess supply.

Proposition 1 *Under an achievement based policy the measure of educated agents is less than* $1/2$ *for any* $\pi \in [0,1]$. *Investment in education is*

$$e^A = \frac{1-2q}{1-q} w,$$

with $q = \frac{1}{2} - \left[\sqrt{\pi^2 w^2 + \frac{1}{4}} - \pi w\right]$. e^A *decreases and* q *increases in* π.

Clearly, $e^A < w < W = e^{LF}$ and forcing integration on the labor market worsens investment incentives. But since firms produce more output under integration than under segregation, rematching has also a positive effect on aggregate surplus. Whether an achievement based policy improves upon the laissez-faire allocation thus depends on whether the gain in output is large enough to offset investment distortions. Aggregate surplus under laissez-faire is $S^{LF} = \pi W^2/2$. An achievement based policy induces total surplus of

$$S^A = \pi e^A \left(2w - \frac{e^A}{2}\right).$$

Therefore the achievement based policy improves on laissez-faire when

$$\underbrace{e^{LF}(2w - W)}_{\text{output gain given LF effort}} > \underbrace{(e^{LF} - e^A)2w}_{\text{output loss given rematch}} - \underbrace{\frac{1}{2}((e^{LF})^2 - (e^A)^2)}_{\text{savings in costs}}. \qquad (1)$$

The LHS captures the surplus added by integration under the achievement based policy keeping investment at its laissez-faire level. The RHS measures the effects on investment: a lower output given the rematch and a savings in cost due to lower incentives. Straightforward calculation shows there exists a unique value of W for which condition (1) holds with equality.

Corollary 2 *Total surplus under an achievement based policy is higher than under laissez-faire if and only if* $W \leq W_0(w, \pi)$.

The cutoff $0 \leq W_0(w, \pi) < 2w$ (as W approaches $2w$ diversity gains vanish, and the incentive losses outweigh them) increases in w and decreases in π (the larger is π, the more likely an ℓ will get w rather than 0, so investment incentives weaken from this insurance effect). Figure 1 depicts the cutoff $W_0(w, \pi)$ as a function of w that separates the areas LF and A when $\pi = 1$.

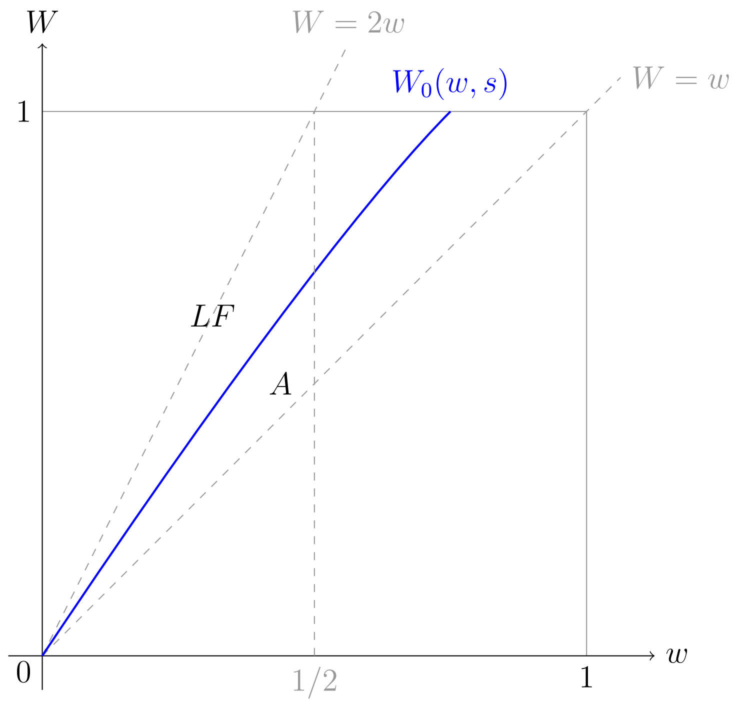


Figure 1: Laissez-faire versus Achievement Based Policies

2.4 Background

The preceding analysis assumed that the measure of agents who are able to invest in education was exogenous. Participation in education is typically subject to individual choice, however. Then labor market returns to education govern the extent of exclusion, since low returns discourage agents from investing. Returns are likely to be affected by individual socio-economic characteristics of agents' neighborhoods or parents, race, or gender, for instance through the cost of accessing education or the profit attained in a firm. These characteristics can be thought of as an agent's type, or *background* b. Suppose that $b \in \{u, p\}$, indicating whether an agent has a *privileged* or an *underprivileged* background. Consistent with the analysis above the measure of agents with background p is π, and the remainder $1 - \pi$ has background u.

Begin with the case that all agents participate in education and firm members' backgrounds b and b' affect production, postponing the case when background affects education cost. Here background may, for instance, measure an individual's set of useful business contacts, or proficiency in languages or social codes. Types in the labor market are now effectively two-dimensional and output in a match of two agents (a, b) and (a', b') is

$$\max\{y(a, a') - 2g(b, b'); 0\}.$$

$g(.)$ denotes a reduction in output depending on the combination of backgrounds, or sets of business contacts. Assume that

$$0 = g(p, p) < g(u, p) = g(p, u) = f < g(u, u) = F.$$

Suppose that $F > w > f$, $F > 2f$, and $W - w > F - f$, i.e. the fixed cost is convex and education matters more for output than background.

When utility is perfectly transferable, under these assumptions a stable allocation exhausts all potential matches of (h, u) and (ℓ, p), and matches agents of the abundant type with (h, p) agents. (ℓ, u) agents segregate and get wage 0; the remaining agents' payoffs depend on scarcity. Denote by e_u the investment of u agents, and by e_p the one of p agents. If (h, u) agents are abundant ($(1 - \pi)e_u > \pi$) the wage for (h, u) agents is the segregation payoff $W - F$. Therefore (h, p) agents get $W + F - f$ and (ℓ, p) agents $2w - W + F - f$ yielding investments $e_u = W - F$ and $e_p = 2(W - w)$.

If $(1 - \pi)e_u < \pi$, (h, u) agents are scarce. When $\pi e_p > \pi(1 - e_p) - (1 - \pi)e_u$, (h, p) outnumber (ℓ, p) agents and get W. Hence, (ℓ, p) agents obtain $2w - W$ and (h, u) agents $W - f$, implying investments $e_u = W - f$ and $e_p = 2(W - w)$. If $2\pi e_p < \pi - (1 - \pi)e_u$, however, (ℓ, p) agents are abundant and get 0. Hence, (h, p) agents get $2w$ and (h, u) agents $2w - f$, yielding $e_p = 2w$ and $e_u = 2w - f$.

Under laissez-faire (h, p) and (ℓ, u) agents segregate as payoffs are monotone in each dimension of the matches' types (a, b). (ℓ, p) and (h, u) agents segregate, since $W - w > F - f$. Hence, p agents invest W, and u agents $W - F$.

Comparing investments in both cases yields the following proposition.

Proposition 2 *Let $F > w > f$, $F > 2f$, and $W - w > F - f$. Compared to the benchmark fully transferable utility allocation laissez-faire induces*

- *under-investment at the bottom, i.e. the underprivileged invest less, if $\pi > (W - f)/(1 + W - f)$, otherwise they invest at the benchmark level,*

- *over-investment at the top, i.e. the privileged invest more, if $\pi < (W - F)/(W - F + 1 - 4(W - w))$, and under-investment at the top if $\pi > (W - f)/(W - f + 1 - 4w)$.*

$W > w + 1/4$ and $\pi > 1/2$ imply simultaneous under-investment at the bottom and over-investment at the top.

Proposition 2 states that the distortion of market prices caused by non-transferable utility may discourage u agents from investing while inducing too much investment by p agents. This is because the return to education depends on both the match and scarcity in the labor market. Under integration returns to education are higher for u agents than under segregation, while the reverse is true for p agents. That is, here a policy of rematching may mitigate investment incentives further by restoring the "correct" returns to education.

Reexamine therefore an achievement based policy in this setting. It exhausts all potential matches between h and ℓ agents. Given this constraint p agents segregate if possible. Similar to above p agents' invest at most w. u agents have strongest incentives when some (ℓ, p) agents remain, which yields at most investment $w - f < W - F$. That is, the downward distortion of incentives by an achievement based policy outweighs any boost in returns to education by rematching, thereby discouraging u even further.

Background Based Policy

Following Ramsey taxation logic a natural way to ameliorate investment distortions is to condition rematching on a characteristic that is less elastically supplied than educational achievement but highly correlated with it, for instance individual background. A *background based* policy integrates u and p agents whenever possible, using uniform rationing when necessary, but otherwise lets agents choose matches freely.

Consider first the case $\pi \leq 1/2$. The underprivileged obtain a privileged match with probability $\pi/(1-\pi)$, and an underprivileged match with probability $(1-2\pi)(1-\pi)$; the privileged get an underprivileged match with certainty. Hence, there are measure π of (u, p) and $1/2 - \pi$ of (u, u) firms; the matching in educational attainment is subject to choice, however.

Denote investments by e_p and e_u for p and u agents. A p agent with achievement h obtains a h match and wage $W - f$ with probability $\min\{e_u/e_p; 1\}$, and otherwise a ℓ match and wage $w - f$. If own achievement is ℓ a p agent obtains a h match and wage $w - f$ with probability $\max\{(e_u - e_p)/(1 - e_p); 0\}$ and otherwise wage 0. Hence, a p agent's investment is bounded above, $e_p < W$.

A u agent with achievement h obtains a (h, u) match and wage $W - F$ with probability $(1 - 2\pi)(1 - \pi)$, a (h, p) match and wage $W - f$ with probability $\min\{e_p/e_u; 1\}\pi/(1-\pi)$, and otherwise a (ℓ, p) match and wage $w-f$. Achieving ℓ a u agent obtains a (h, p) match and wage $w - f$ with probability $\max\{(e_p -$

$e_u)/(1-e_u);0\}\pi/(1-\pi)$, and otherwise a wage 0. Calculations reveal that a u agent's investment is bounded below, $e_u > W - F$.

Hence, a background based policy encourages the underprivileged to invest while discouraging the privileged, since segregation payoffs for high achievers, which are high for the privileged, but low for the underprivileged, are no longer attained with certainty. Indeed for $\pi \neq 1/2$ it must be the case that $e_u \neq e_p$, since segregation payoffs of (h,p) and (u,p) agents (W and $W-F$) differ. That is, the measures of high achievers with u and p background are not proportional, so that uniform rationing generates some fully integrated firms whose members differ in both achievement and background. Therefore a background based policy ensures some integration on the achievement dimension thus reaping some gains from rematching. This intuition extends to the case $\pi > 1/2$ and gives raise to the following proposition, see Appendix for the full proof.

Proposition 3 *Let $F > w > f$, $F > 2f$ and $W - w > F - f$. Then*

- *the privileged invest $e^{LF} = W$ and the underprivileged $e^{LF} = W - F$ under laissez-faire,*

- *the privileged invest $e^A \leq w$ and the underprivileged $e^A \leq w - f < W - F$ under an achievement based policy,*

- *the privileged invest $W - F < e^B < W$ and the underprivileged $W - F < e^B < W$ under a background based policy.*

This reflects the encouragement effect of affirmative action discussed by Coate and Loury (1993), inducing the underprivileged to invest more now that they expect a significant return because of the policy. Note though, that here, as elsewhere, the privileged agents' incentives are reduced. Indeed this is a general insight from our analysis: the group not favored by the policy has reduced incentives; the group favored by the policy may have improved incentives, as in this case, or not, as in the case of achievement based policy.

3 The Schooling Stage

Indeed most education investments are taken not in solitude but rather in a social environment where the behavior of an agents' peers influences own behavior. This may be by way of social norms and role models, learning spill-overs in class, or pure cost externalities. For the purpose of modeling we focus on the last and assume that agents are heterogenous in cost of accessing education, which depends on an agents' peers at school. We focus on heterogeneity in cost of access to education rather than in marginal cost of acquiring education. Whereas marginal cost of education may reflect individual ability, access cost captures an agent's socioeconomic background. Let therefore $g(b, b')$, as defined above, denote an agent's fixed cost of education investment. For the remainder of the paper the cost $g(.)$ is incurred at the investment stage, and depends on that agent's investment environment, or *club* (b, b').[5] A club is best understood as a school, but could also capture e.g. parents' neighborhood choice.

In the following we limit our attention to cases that satisfy some parametrical assumptions on the fixed cost $g(.)$.

Assumption 1 (Fixed Cost) $f < (W - w)^2 < W^2/2 < F < 2w^2$.

This assumption ensures that agents with cost f find it profitable to invest when matching into integrated firms, and that high cost F agents find it optimal to invest when paid the full social benefit of turning a (ℓ, ℓ) firm into a (h, ℓ) firm.

3.1 Fully Transferable Utility Benchmark

Derive now the fully transferable utility allocation as a benchmark for the full model. As shown above on the labor market h agents get wage $w(h) \in [W, 2w]$ depending on the scarcity of h versus ℓ agents. Investment is $e^{TU} = 2(w(h) - w)$ giving payoff $2(w(h) - w)^2 + 2w - w(h) - g(b, b')$. When utility is transferable, u agents may compensate p agents for the cost externality. Integrated (u, p) clubs are stable if the joint payoff exceeds the sum of segregation payoffs:

$$\begin{aligned} (w(h) - w)^2 &> f \text{ if } F > 2(w(h) - w)^2 + 2w - w(h) \text{ and} \\ F &> 2f \text{ if } F < 2(w(h) - w)^2 + 2w - w(h). \end{aligned}$$

Hence, Assumption 1 implies integration both on the labor market and at the schooling stage. For investments two interesting cases arise as stated in the following proposition.

Proposition 4 *Suppose Assumption 1 holds. When utility is fully transferable both schools and the labor market integrate. Moreover,*

[5] This differs from the case above, where background affected productivity, in that each dimension is matched separately, and fixed cost are incurred in between. For a comparison of these regimes see Gall et al. (2006).

(i) if $\min\{2\pi;1\}2(W-w) > 1/2$*,* $s = \min\{2\pi;1\}$*, investments are* $e = 2(W-w)$ *and* $q > 1/2$*,*

(ii) otherwise $q = \min\{2w;1/2\}$ *and*

- *if* $2w > 1/2$ *and* $\pi \geq 1/2$*,* $s = 1$ *and investments are* $e = 1/2$*,*
- *if* $2w > 1/2$ *and* $\pi < 1/2$*,* $s = 2\pi + \max\{0; 1/2 - 2\pi\sqrt{2F}\}$ *and investments are* $e = \min\{\frac{1}{4\pi}; \sqrt{2F}\}$*.*
- *if* $2w \leq 1/2$*,* $s = 1$ *and investments are* $e = 2w$*.*

Proof: In Appendix.

In case (i) social returns to education are high enough for $q > 1/2$ when all agents in (u,p) and (p,p) clubs, while (u,u) agents do not. In case (ii) social returns are high enough to induce all agents, even those in (u,u) clubs, to invest when $q < 1/2$, but not when $q > 1/2$. Hence, $q = 2w$ if all agents invest but $w < 1/4$. Otherwise $q = 1/2$ and the market price adjusts to make (u,u) agents indifferent between investing or not.

That is, if the number of privileged agents and the value added in (h,h) firms, $W - w$, are sufficiently great, the underprivileged underinvest and the privileged overinvest under laissez-faire compared to the benchmark allocation. Otherwise all agents underinvest, mirroring Proposition 2. This means the effects of nontransferable utility vary with the characteristics of the economy. Abundance of the underprivileged and a technology that values skilled labor input best describes an industrialized country, whereas the reverse seems true in developing economies. Maintaining this interpretation, our results indicate that non-transferable utility exacerbates inequality of opportunity in industrialized countries by discouraging underprivileged agents, while the discouragement effect is universal for developing economies.

3.2 Nontransferable Utility

Suppose that utility is strictly nontransferable also at the school stage. That is, payoffs from school choice (b,b') are given by the resulting allocation on the labor market. This applies well when for instance school choice requires monetary investment (either in form of fees or house prices) and agents differ in capital cost.

Laissez Faire

Both the labor market and the school stage operate under laissez-faire. We established above that given educational attainments h and ℓ the labor market segregates. Hence, an agent in club (b, b') obtains payoff $\max\{W^2/2 - g(b, b'); 0\}$. Indeed an allocation with integrated (u, p) clubs cannot be stable when utility is (sufficiently) nontransferable. This is because p agents strictly prefer a (p, p) to a (u, p) school as $f > 0$, so that any two p agents matched into (u, p) clubs have a profitable joint deviation. Thus schools segregate under laissez-faire and surplus is $S^{LF} = \pi W^2/2$ as above. That is, under nontransferable utility there is too much segregation whenever $4f < W^2$.

Hence, in the full model associational redistribution could be beneficial in the labor market, at school, or at both stages. The analysis above has shown that using an achievement based policy in the labor market may increase aggregate surplus compared to laissez-faire at the cost of a downward distortion of investment incentives. Moreover, a background based policy improves the alignment of private to social investment incentives, when background affects the firm's profit directly. Yet even in the present setting, when background does not enter the firm's profit function directly, a background based policy may dominate an achievement based policy. This is since conditioning on a characteristic not subject to individual choice but highly correlated with it may mitigate incentive distortions.

Background Based Labor Market Policy

Let us therefore consider the use of only a background based policy in the labor market that integrates u and p agents whenever possible, using uniform rationing when necessary, but otherwise lets agents choose matches freely.

As above in case $\pi \leq 1/2$ each u agent is assigned a p match with probability $\pi/(1-\pi)$ and u otherwise, while p agents obtain a u match with certainty. As u agents attain ℓ under segregation at school, a p agent with high achievement is matched into an integrated firm (h, ℓ) obtaining wage w for sure, as under an achievement based policy. Privileged low achievers, however, have now probability 0 of matching into a (h, ℓ) firm. This reduces their expected payoff compared to an achievement based policy. Hence, when $\pi < 1/2$, a background based policy induces a redistribution towards u agents (having a higher chance to obtain wage w) and stronger incentives for p agents. As the measure of firms (h, ℓ) is $\pi e^B > \pi e^A$, a background based policy increases total output and dominates an achievement based policy when $\pi \leq 1/2$.

Hence, it will generate higher aggregate surplus than laissez-faire also in the neighborhood of the curve $W_0(w, \pi)$. Similar to above laissez-faire induces better incentives, but less efficient matches. One can show that there is a cutoff value $W_2(w, \pi) > W_0(w, \pi)$ such that laissez-faire yields higher total surplus than a background based policy if and only if $W > W_2(w, \pi)$. For $\pi \leq 1/2$, $W_2(w, \pi) = \sqrt{3}w$.

When $\pi > 1/2$, a privileged agent optimally invests

$$e^B = \underbrace{\frac{1-\pi}{\pi}w}_{\text{match with } u \text{ agent}} + \underbrace{\frac{2\pi - 1}{\pi}W}_{\text{match with a high achieving } p \text{ agent}} . \tag{2}$$

As $e^A < w$ we have $e^B > e^A$, and a background based policy induces redistribution towards the underprivileged and stronger incentives for the privileged as in the case above.

Yet now total surplus is not necessarily higher under a background based policy. As $\pi > 1/2$, some p agents must form (p, p) matches. Since a background based policy does not condition on achievement, these agents segregate as under laissez-faire. Hence, there is a positive measure of (h, h) and (ℓ, ℓ) firms, which is inefficient from a surplus point of view. That is, for $\pi > 1/2$ a background based policy induces better incentives and a less efficient matching than an achievement based policy. An argument similar to the one in Corollary 2 yields a cutoff $W_1(w, \pi)$ such that a background based policy is preferable to one based on achievement if $W > W_1$, and the reverse is true if $W < W_1$. Since the amount of mismatch increases in π, so does $W_1(w, \pi)$, which also increases in w. The following lemma summarizes properties of the cutoff values.

Lemma 2 *There are functions $W_1(w, \pi)$, $W_2(w, \pi)$ with the properties*

- $2w \geq W_2(w, \pi) > W_0(w, \pi) > W_1(w, \pi) \geq w$,
- $W_1(w, \pi), W_2(w, \pi)$ *are increasing in w and in π,*
- $W_1(w, \pi) = w$ *for $\pi \leq 1/2$, and*
- $W_2(w, \pi) = \sqrt{3}w$ *for $\pi \leq 1/2$,* $\lim_{\pi \to 1} W_2(w, \pi) = 2w$,

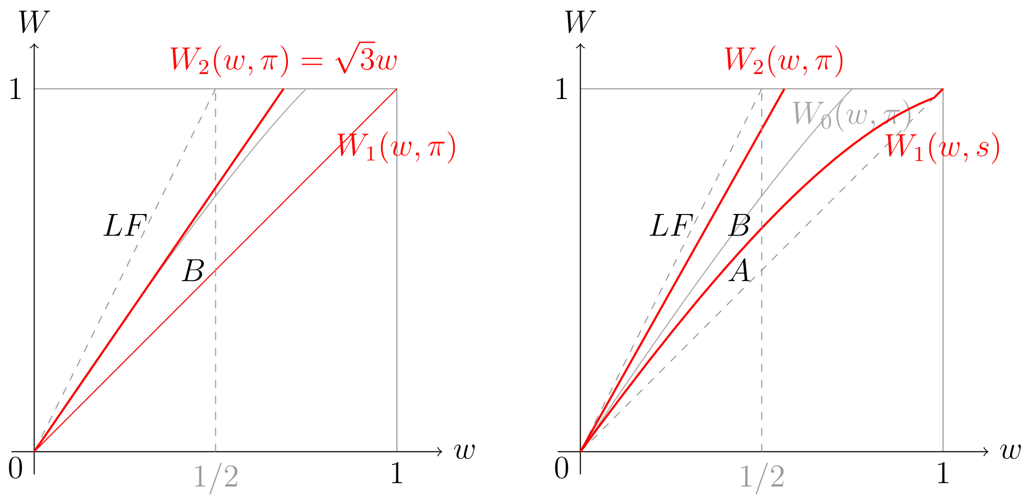
When compared to laissez-faire in case $\pi > 1/2$ a background based policy provides worse incentives but a more efficient matching, as above. The following proposition gives the surplus maximizing policy depending on parameters, and is illustrated in Figure 2.

Proposition 5 *The surplus maximizing policy is*

(i) Laissez-faire when $W \geq W_2(w, \pi)$*,*

(ii) Background based when $W \in (W_2(w, \pi), W_1(w, \pi))$*, and*

(iii) Achievement based when $W < W_1(w, \pi)$*.*



(a) $\pi < 1/2$

(b) $\pi > 1/2$

Figure 2: Laissez-faire, Achievement, and Background Based Policies

School Integration Policy

As nontransferable utility may induce mismatch at school, in form of too much segregation, there may be a role for associational redistribution at school as well. For instance, a policy of school integration that forces agents to invest in integrated (u, p) environments should raise aggregate surplus if bringing in u agents is cost efficient. A *school integration* policy matches u to p agents at school whenever possible, using uniform rationing when necessary. A prime example of this as practiced in the U.S. is "busing." More contemporaneously, policies determining the diversity of schools in terms of pupils' backgrounds vary substantially across countries. One indicator of this is the extent of sorting pupils into ability stratum, a policy called tracking. Pupils' ages when first sorted ranges from 10 in Austria and Germany to 16 and above in the US or most of Scandinavia (see Table 5.20, OECD, 2004). Empirical evidence indicates that the degree of tracking affects the dependence of students' educational attainments on parental background, similar to school integration.[6]

Under school integration, if $\pi \leq 1/2$ the measure of (u, p) clubs is 2π, and the one of (u, u) clubs is $(1-\pi)/2$; otherwise there are measures $1-\pi$ of (u, p) and $\pi - 1/2$ of (p, p) clubs. Let s denote the measure of agents with strictly positive investment in equilibrium. Supposing $s > 0$,[7] laissez faire labor market wages imply investments $e^I = W$ in (p, p) clubs, $e^I = 0$ in (u, u) clubs, and $e^I = W$ if $W^2 > 2f$ and otherwise 0 in (u, p) clubs. Therefore $s^I = \min\{1; 2\pi\}$ if $W^2 > 2f$ and otherwise $s^I = \max\{0; 2\pi - 1\}$. Aggregate surplus under school integration is

$$S^I = sW^2/2 - (s - \max\{2\pi - 1; 0\})f.$$

Therefore

$$S^{LF} > S^I \Leftrightarrow W^2 < 4f.$$

This does not depend on the DD property ($W < 2w$). Hence, school integration may restore the benchmark allocation at the schooling stage under fully transferable utility when $4f < W^2$.

To give a specific example, Meghir and Palme (2005) study a schooling reform in Sweden that was implemented around 1950. The reform increased compulsory schooling by three years, abolished tracking after grade 6, and imposed a nationally unified curriculum. That is, the policy aimed at decreasing school segregation in backgrounds. The policy change increased education acquisition (beyond the new compulsory level for highly able pupils) and labor income for individuals whose fathers had low education, while it did not significantly change education acquisition and lowered wage income for individuals whose father had high education.[8]

[6]See e.g. Schütz et al. (2008), Brunello and Checchi (2007), Ammermüller (2005) who find that the strength of this dependence depends positively on earlier start of tracking, and number of tracks or private schools.

[7]If $\pi < 1/2$ and $w^2 < 2f$ there is another equilibrium where nobody invests. It is not robust to school integration that allows measure $\pi > \epsilon > 0$ of p agents to segregate.

[8]Segregation at school may not only apply to sorting of students. Teachers may share a preference for safe schools and motivated students, possibly to an extent that cannot be compensated by public salaries (see Hanushek et al., 2004).

4 Policies Affecting Both Stages

An important practical concern is whether effectiveness of school integration depends on the labor market policy in place. Hence, we are interested in whether associational redistribution on the labor market and at the school stage may act as complements or substitutes, that is whether they reinforce or cancel each other. Two major concerns may arise: first, school integration raises the access cost of p agents, which may lead to discouragement due the investment distortion under an achievement based policy. Second, integrating schools weakens the link between background and educational outcome, thus reducing the effectiveness of a background based policy.

4.1 Achievement Based Policy and School Integration

Suppose that an achievement based policy is used on the labor market in conjunction with integration at school. As above optimal effort choice satisfies

$$e^{AI} = \frac{1-2q}{1-q}w, \tag{3}$$

if $q \leq 1/2$, where q is the measure of h agents. e^{AI} depends on s^{AI} via q, and indeed $e^{AI}(s) = e^{A}(s)$, but $s^{A} = \pi$ whereas s^{AI} is endogenous. By Proposition 1 $q < 1/2$. Aggregate surplus under a combination of achievement based and school integration policies is

$$S^{AI} = s^{AI}e^{A}(s^{AI})\left(2w - \frac{e^{A}(s^{AI})}{2}\right).$$

An agent in a (u, p) schooling environment invests if

$$\left(\frac{1-2q}{1-q}\right)^2 \frac{w^2}{2} > f.$$

Investments of agents in (u, p) clubs depend on q and determine s^{AI}, which in turn affects q. If f is small enough, agents in (u, p) clubs invest, otherwise they do not, with adverse consequences for total surplus.

Proposition 6 *Under school integration and an achievement based policy*

(i) in case $\sqrt{2f} < e^{A}(\min\{2\pi; 0\})$*:* $e^{AI} < e^{A}$, $q^{AI} > q^{A}$, $s^{AI} = \min\{2\pi; 1\} > s^{A}$, *and* $S^{AI} > S^{A}$,

(ii) in case $\sqrt{2f} > e^A(\max\{2\pi-1;0\})$*:* $e^{AI} > e^A$, $q^{AI} < q^A$, $s^{AI} = \max\{2\pi - 1;0\} < s^A$, *and* $S^{AI} < S^A$.

Proof: In Appendix.

There may arise the case, e.g. when $\pi \leq 1/2$ and $w^2/2 < f < (W-w)^2$, that school integration induces zero investments, given an achievement based policy on the labor market, since incentives to invest are depressed.

4.2 Background Based Policy and School Integration

Turn now to a combination of a background based policy on the labor market and integration at school. Then measure $2\min\{\pi;1-\pi\}$ of agents are matched into (u,p) clubs with access cost f. The labor market policy requires all possible matches between u and p agents to be exhausted, using uniform rationing if necessary. Given this constraint agents are free to match. If all agents in (u,p) schools choose the same investment (i.e. the equilibrium is symmetric in fixed cost type), the resulting measures of h agents with background u and h agents with background p will always enable full segregation in achievement in accordance with the policy.[9] This replicates the outcome under laissez faire on the labor market and school integration.

Proposition 7 *Let a school integration policy be in place. Then, in a symmetric equilibrium, a background based policy yields the laissez faire allocation on the labor market,*

$$e^{BI} = W = e^{LF} = e^I, s^{BI} = \min\{2\pi;1\} = s^I, S^{BI} = S^I.$$

Proof: In Appendix.

Hence, school integration may render background based policies completely obsolete. This is since background carries less information about achievement as schools become more integrated. On the other hand, school integration encourages investment of underprivileged agents and in effect generates equality of opportunity by equalizing access cost. Hence, under school integration the role for associational redistribution on the labor market is much reduced, and limited to rematching.

[9] Asymmetric equilibria cannot in general be precluded, see Appendix for a discussion.

4.3 Club Based Policies

When it is infeasible to learn agents' cost types, for instance due to legal or informational constraints, a policy of associational redistribution may nevertheless be able to condition on the school environment or club (b, b'). For instance, this could be the socio-economic characteristics of neighborhoods individuals live in, or the performance rank of the school attended. A notable example is the Texas Top 10 Percent law used to admit students into the University of Texas. Others include measures of placing students from disadvantaged neighborhoods or schools in firms, e.g. school-to-work-policies like the School-to-work Opportunities Act in the US.

Such a *club based* policy could work as follows in our model:

- Agents are free to choose schools,
- alumni of (p, p) schools must match with alumni of (u, u) schools whenever possible, using uniform rationing when necessary,
- alumni of (u, p) schools are unrestricted by the planner in choosing a partner on the labor market.

Denote the measure of (u, u) pupils by s_u, and the one of (p, p) pupils by s_p. Then (u, p) pupils have measure $1 - s_u - s_p$. Suppose first that all schools are segregated, $s_u = 1 - \pi$ and $s_p = \pi$. Then a p agent matches to a u agent with probability $(1 - \pi)/\pi$ if $\pi > 1/2$ and with certainty if $\pi \leq 1/2$. Since $F > W^2/2$ agents in (u, u) environments do not invest and p agents solve

$$\max_e ew - \frac{e^2}{2} \text{ if } \pi \leq 1/2 \text{ and}$$
$$\max_e e\left(\frac{s_u}{s_p}w + \left(1 - \frac{s_u}{s_p}\right)W\right) - \frac{e^2}{2} \text{ if } \pi > 1/2.$$

Interior solutions satisfy $e^C = W - (s_u/s_p)(W - w)$ if $\pi > 1/2$, and $e^C = w$ otherwise. Hence, $e^C = e^B$ if $s_p = \pi$ and $s_u = 1 - \pi$, and a club based policy coincides with a background based policy when schools segregate.

Let us focus on the case $\pi > 1/2$.[10] To verify whether school segregation is an equilibrium, suppose a pair of agents match into a (u, p) school. In the

[10]Otherwise some integration remains an equilibrium outcome, although multiplicity of equilibria becomes an issue, see the appendix.

labor market they may segregate in education outcome (h agents have positive measure as $\pi > 1/2$), and their investments solve

$$\max_e eW - e^2/2 - f.$$

The optimal choice $e = W$ generates payoff $W^2/2 - f > 0$, so (u,p) agents invest. Agents segregate into schools if payoffs are higher in segregated than in integrated schools for p or u agents, that is if

$$(e^C)^2 > W^2 - 2f \text{ or } 2e^C w > W^2 - 2f,$$

since (u,u) agents expect payoff $e^C w$ due to uniform rationing. The first condition implies the second as $2w > e^C$. Therefore schools segregate and background and club based policies coincide if and only if

$$2f > \frac{1-\pi}{\pi} 2w(W-w) - W(2w - W). \tag{4}$$

If this is not the case, then p agents prefer (u,p) clubs when $s_p = \pi/2$. Therefore $s_u + s_p < 1$ if (4) does not hold. Let now $s_u = 0$. A (u,p) agent can segregate in education, and, assuming (p,p) agents invest (generating a positive measure of h agents), solves

$$\max_e eW - e^2/2 - f.$$

As $W^2 > 2f$, (u,p) and (p,p) agents invest. When deviating to segregated schools, u agents obtain ew, p agents obtain $W^2/2$. Since $W^2/2 > W^2/2 - f$ all p agents in (u,p) clubs can profitably deviate to a (p,p) school. Hence, $s_u > 0$ and $s_p > 0$.

(u,p) agents segregate on the labor market and invest $e = W$. Incentive compatibility for school sorting binds for p agents, so that s_p and investment of (p,p) agents leaves them indifferent between (u,p) and (p,p) schools, see Appendix for details. This implies the following proposition.

Proposition 8 *If* $2f < \frac{1-\pi}{\pi} 2w(W-w) - W(2w-W)$*, a club based policy*
(i) induces integration at school, $s_p + s_u < 1$ *and* $s_u > 0$*, and on the labor market, as the measures of* (ℓ,ℓ)*,* (ℓ,h)*, and* (h,h) *firms are all positive,*
(ii) generates investments $e = 0$ *in* (u,u)*,* $e = W$ *in* (u,p) *and* $e = (W^2 - 2f)/(2w) > e^B$ *in* (p,p) *clubs.*
Otherwise club based and background based policies coincide.

The next proposition evaluates welfare under a club based policy.

Proposition 9 *Let $W^2 > 6f$. There exists $\pi^* > 1/2$ such that for all $1/2 \leq \pi < \pi^*$ a club based policy dominates all other labor market policies (achievement based, background based, and laissez faire) if and only if*

$$W^2 - 2f > 2w\left(W - \frac{1-\pi}{\pi}(W-w)\right).$$

Proof: In Appendix.

That is, if access cost in integrated schools is small enough and the measure of privileged close to 1/2, a club based policy dominates other labor market policies whenever it induces desegregation at school.[11] It successfully trades off investment incentive and output effects from rematching. Negative incentive effects of integration on the labor market are curbed by conditioning on club membership rather than on achievement. Negative effects of school integration due to reducing quality of screening are counteracted by incentive compatibility of the sorting equilibrium, which requires positive measures of (u, u), (u, p), and (p, p) clubs. Finally, a club based policy reaps at least part of the benefits from rematching both at school and on the labor market, since the all firm and club constellations have positive measure.

Proposition 9 also applies when property DD does not hold, i.e. $2w < W$. Then segregation on the labor market maximizes output all else equal.[12] Also in this case, when fixed costs in integrated schools are low enough for a club based policy to induce school integration, this policy dominates laissez-faire.

An illuminating example of club based policies is admission of high school graduates to public universities in Texas. In late 1996, Texas state universities abolished affirmative action based on race in response to the Fifth Circuit Court decision in *Hopwood vs. Texas.* In 1997 the Texas Top 10 Percent law was instituted with the stated aim to preserve minority attendance rates. This scheme guarantees automatic admission to Texas state universities for students who graduate among the best ten percent of their class. Since Texan high schools were highly segregated this was expected to counteract adverse effects of abolishing affirmative action to campus diversity, tacitly assuming that the policy change did not affect composition. Kain et al. (2005) report that

[11] Indeed it dominates a background based policy unconditionally.

[12] Also under fully transferable utility the labor market segregates in education, and wages coincide with those under strictly nontransferable utility. Hence, this result requires (sufficiently) nontransferable utility at the schooling stage, but not on the labor market.

> Hopwood had a devastating effect on minority enrollment in Texas selective public universities, reducing the African-American and Hispanic share of entering classes by 37 percent and 21 percent between 1996 and 1998.

That is, after about two decades of affirmative action in Texas its removal triggered a sudden reversal to segregation. This may indicate that affirmative action policies were ineffective in changing beliefs, or that segregation in higher education was not entirely belief-based. Kain et al. (2005) further conclude that the Texas Top 10 percent law was not effective in preserving campus diversity since the top slots were disproportionally taken by non-minority students. Long (2004) confirms both observations in a broader study covering US-wide substitution of affirmative action by high school quotas.

Parents appear to have reacted to incentives, as Cullen et al. (2006) report some evidence of strategic sorting by good students into worse peer-groups in Texas. This is consistent with our model where a club based policy may induce a rematching of schools. If diversity at school is desirable from a social planner's point of view, the Texas Top 10 Percent Law seems an excellent example of unintended, yet beneficial consequences.

5 Conclusion

We presented a framework to analyze policies of associational redistribution on the labor market and at school. The framework imposes strictly nontransferable utility serving to focus on the interaction of matching patterns and investment incentives. It remains silent, however, about another source of inefficiencies when utility is transferable, but not perfectly so. Then competition may require inefficient sharing of surplus (see e.g. Legros and Newman, 2008) which in turn affects investment incentives. Pursuing this topic appears to be an important task for future research.

In the present approach policies aim at replicating the fully transferable utility matching outcome, that is integration, as a benchmark. In a more complex derivation of nontransferable utility, nontransferabilities may affect the optimal matching, however. See Gall et al. (2008) for an example when information rents decrease in the scope of the project, so that the optimal matching involves integration when there is asymmetric information generating nontransferabilities, but segregation under perfect information.

Labor market policies need to trade off output efficient sorting and provision of adequate incentives for pre-match investments. Conditioning labor market re-matching on observable information not subject to individual choice, such as background, appears beneficial when it is linked to education outcome. Early stage intervention, i.e. at school, does not distort incentives and provides benefits when integrating schools is cost efficient. In that sense early stage policies are more effective than later stage policies.

Earlier and later stage policy are interdependent, however. School integration may limit the informational content provided by individual background and reduce the effectiveness of screening, rendering background based policies obsolete. When an achievement based policy is used on the labor market school integration may discourage investment due to low returns to education. Moreover, optimal policies may depend on characteristics of the economy. For instance, if privileged agents are scarce, a background based policy dominates an achievement based policy. This does not hold for economies where the privileged abound, suggesting that the use of achievement based policies should be restricted to developed economies.

Finally, we identify a labor market policy that looks promising in terms of trading off incentive provision and efficient sorting: a club based policy rematches the labor market conditioning on individual school choices. It yields some integration both on the labor market and at school while inducing higher investments than other policies. This result is particular encouraging since there is no reason to expect this policy to be optimal. The nature of optimal mechanisms of associational redistribution in sequential assignment markets when utility is nontransferable remains an open question.

A Mathematical Appendix

Some Transferability

To see that our results are robust to admitting some transferability in the labor market assume first that surplus is shared equally in firms, but agents have quasilinear utility and can make side payments up to b. This can be thought of cash endowments in the absence of credit markets. Integration under Assumption DD is possible if

$$w + t \geq W \text{ and } w - t \geq 0,$$

where $t \in [-b, b]$ is a side payment from a ℓ agent to a h agent. Hence, if $b < W - w$ utility is sufficiently transferable to ensure that the labor segregates.

Another cause for nontransferable utility is moral hazard within a firm. To illustrate this suppose that output is stochastic and occurs only with probability $\rho(x, x')$, which depends on effort x and x' chosen by the firm members. Suppose that effort x induces a utility cost of $x^2/2$ and that

$$\rho(x, x') = (x + x')^\delta,$$

where $\delta \in [0, 2)$ is a parameter indicating the responsiveness of the success probability to effort. Let output depend on firm members' achievements through $y(a, a')$ and assume that expected surplus in firm is

$$(x + x')^\delta y(a, a')^{\frac{2-\delta}{2}} - (x^2 + (x')^2)/2.$$

The exponent $(2 - \delta)/2$ serves to compensate scale effects from larger projects $y(a, a')$ on effort choice. This ensures that desirability of integration is governed by the properties of $y(.)$ independent of δ. Both partners in the match can contract on the sharing of surplus ex ante, at the matching stage. Denote the individual shares by s and s' with $s + s' = 1$. Given a sharing rule s optimal individual effort choice satisfies for $\alpha > 0$

$$x = s\delta(x + x')^{\delta-1} y(a, a')^{(2-\delta)/2}, \tag{5}$$

and analogously for x'. This yields $(1 - s)x = sx'$ and the solutions

$$x(s) = s\delta^{\frac{1}{2-\delta}} y(a, a')^{1/2} \text{ and } x'(s) = (1 - s)\delta^{\frac{1}{2-\delta}} y(a, a')^{1/2}.$$

The Pareto frontier is then given by the solution to

$$\max_{s \in [0,1]} \left(\delta^{\frac{\delta}{2-\delta}} - \frac{s\delta^{\frac{2}{2-\delta}}}{2} \right) s y(a, a') \text{ s.t. } \left(\delta^{\frac{\delta}{2-\delta}} - \frac{(1-s)\delta^{\frac{2}{2-\delta}}}{2} \right) (1-s) y(a, a') \geq u_b.$$

Either the constraint binds, and

$$1 - s = \frac{1}{\delta} - \frac{\sqrt{\delta^{2\frac{\delta-1}{2-\delta}} - \frac{2u_b}{y(a,a')}}}{\delta^{\frac{1}{2-\delta}}}. \tag{6}$$

This expression defines $s(u_b)$. Otherwise maximizing u_a does not constrain u_b, so that $s = \frac{1}{\delta}$. Note that this is only possible for $\delta > 1$. Using the expression (6) the Pareto frontier can be written as

$$u_a(u_b) = s(u_b)\delta^{\frac{2}{2-\delta}} \left(\frac{1}{\delta} - \frac{s(u_b)}{2} \right) y(a, a'), \tag{7}$$

whenever the constraint binds, i.e. the following condition holds:

$$\frac{3-\delta}{2}(\delta-1)\delta^{2\frac{\delta-1}{2-\delta}}y(a,a') \leq u_b, \tag{8}$$

and otherwise

$$u_a(u_b) = \delta^{2\frac{\delta-1}{2-\delta}}\frac{y(a,a')}{2}. \tag{9}$$

Note that $u_a(u_b)$ defined by (7) is concave in u_b.

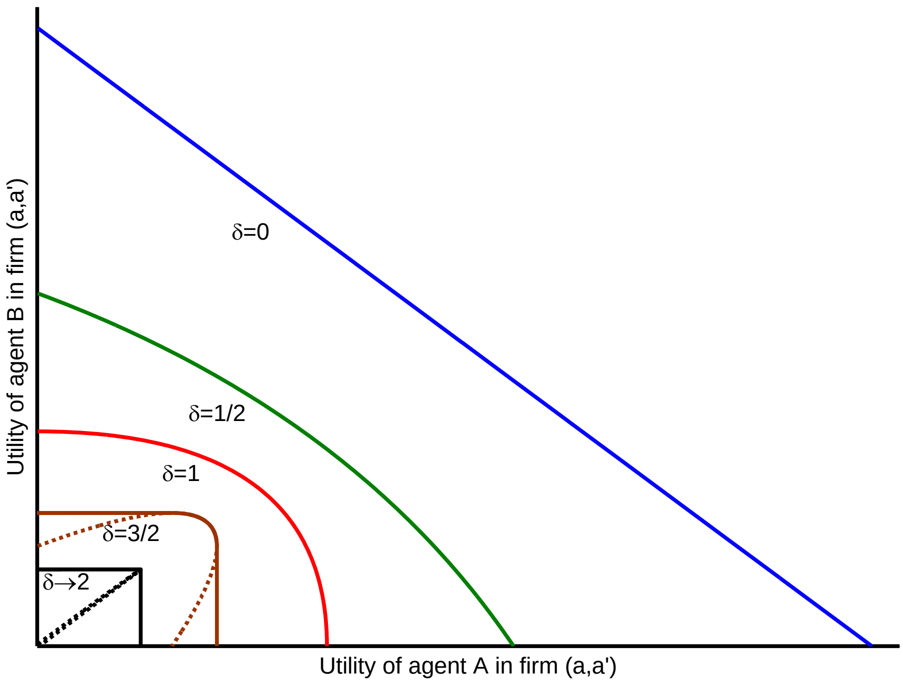


Figure 3: Partners' utilities corresponding to choices of s given δ.

Figure 3 shows the Pareto frontiers (solid lines) in partnership problems associated to different values of δ. In case $\delta \leq 1$ agents' utilities corresponding to effort choices $x(s)$ and $x'(s)$ for all $s \in [0,1]$ determine the Pareto frontier given by (7). Clearly, for $\delta = 0$ it is optimal to set $x = 0 = x'$, so that in this case utility is perfectly transferable (the Pareto frontier is a straight line with slope -1), since $\rho(x, x') = 1$ independently of effort choice and thus of s. As δ increases, so does the curvature of the Pareto frontier, decreasing transferability by making compensations more costly in terms of total surplus. For $\delta > 1$ the constraint (8) may not bind, so that the locus of agents' utilities that can be reached by choice of $s \in [0,1]$ may be bend backwards, as depicted by the dashed lines. In that case the points on the Pareto frontier are given by (9).

Segregation payoffs in a firm (a, a) share the surplus equally and are

$$\underline{u}(a) = \delta^{\frac{2}{2-\delta}} \left(\frac{1}{2\delta} - \frac{1}{8} \right) y(a, a).$$

Note that equal incentives (and effort provision) is efficient (see Ray et al., 2007) so that desirability of integration depends indeed on the properties of $y(.)$. Integration will maximize total surplus if and only if $2y(a, a') > y(a, a) + y(a', a')$, which is implied by Assumption DD.

A necessary condition for integration is that wages in a firm (a, a') can be chosen to exceed segregation payoffs of both firm members. Letting $a > a'$, this is expressed in the following condition:

$$u_a(\underline{u}(a')) \geq \underline{u}(a). \tag{10}$$

The following proposition states that there always is an effort provision technology characterized by parameter δ such that integration is not sustainable in equilibrium, i.e. (10) fails.

Proposition 10 *For any $0 \leq y(a', a') < y(a, a') < y(a, a)$ there exists a $\delta^* < 2$ such that condition (10) fails for all $\delta \in (\delta^*, 2)$. If $y(a', a') = 0$ and $y(a, a)/y(a, a') > 4/3$, $\delta^* = 1$.*

Proof: If condition (8) holds, (10) can be rewritten using (7) to yield

$$s(\underline{u}(a'))\delta^{\frac{2}{2-\delta}} \left(\frac{1}{\delta} - \frac{s(\underline{u}(a'))}{2} \right) y(a, a') \geq \delta^{\frac{2}{2-\delta}} \left(\frac{1}{2\delta} - \frac{1}{8} \right) y(a, a), \tag{11}$$

and otherwise

$$\delta^{2\frac{\delta-1}{2-\delta}} \frac{y(a, a')}{2} \geq \delta^{\frac{2}{2-\delta}} \left(\frac{1}{2\delta} - \frac{1}{8} \right) y(a, a).$$

This can be rewritten as

$$\bar{y} \leq \frac{1}{\delta - \delta^2/4}, \tag{12}$$

where $\bar{y} = y(a, a)/y(a, a') > 1$. Hence, a sufficient condition for (12) to fail is $\delta - \delta^2/4 > 1/\bar{y}$. By continuity there exists $\delta_0 < 2$ such that this is the case for all $\delta \in (\delta_0, 2)$. Indeed $\bar{y} > 4/3$ ensures that this is the case whenever (8) fails. (8) can be rewritten in case $u_b = \underline{u}(a')$ as

$$\frac{(3 - \delta)(\delta - 1)}{\delta - \delta^2/4} \leq \underline{y},$$

where $\underline{y} = y(a', a')/y(a, a') < 1$. As above, by continuity there exists $\delta_1 < 2$ such that the above condition fails for all $\delta \in (\delta_1, 2)$. Hence, for any technology $\bar{y} > 1 > \underline{y}$ there is a $\delta^* = \max\{\delta_0; \delta_1\}$ sufficiently close to 2 such that both (8) and (12) fail for all $\delta \in (\delta^*, 2)$. For $y(a', a') = 0$ we have $\delta_1 = 1$ and $\delta^* = \delta_0$. □

Turning back to (11) for the sake of completeness and solving for $s(.)$ yields

$$s(\underline{u}(a')) \geq \frac{1}{\delta}\left(1 - \sqrt{1 - \bar{y}\left(\delta - \frac{\delta^2}{4}\right)}\right),$$

By (6)

$$s(\underline{u}(a')) = \frac{\delta - 1}{\delta} + \frac{\sqrt{\delta 2^{\frac{\delta-1}{2-\delta}} - \frac{2\underline{u}(a')}{y(a,a')}}}{\delta^{\frac{1}{2-\delta}}}.$$

Condition (11) takes care of nonnegativity of the term under the root. Combining the last two expressions yields a necessary condition for integration

$$\sqrt{1 - \underline{y}\left(\delta - \frac{\delta^2}{4}\right)} + \sqrt{1 - \bar{y}\left(\delta - \frac{\delta^2}{4}\right)} \geq 2 - \delta.$$

Since $\underline{y} \geq 0$, the above condition fails whenever $\delta < 1$ and

$$\bar{y} > \frac{2 - \delta}{1 - \delta/4}.$$

Proof of Lemma 1

When maximizing expected utility $u = ew(h) + (1 - e)w(\ell) - \frac{e^2}{2}$, a necessary condition for optimal investment is $e^{TU} = 2(w(h) - w)$.

We have established in the text that if $q > 1/2$, agents with education h are abundant and obtain wage $w(h) = W$, agents with ℓ obtain $w(\ell) = 2w - W$. Hence, $e^{TU} = 2(W - w)$ and the realized $q = \pi 2(W - w)$, which is greater than 1/2 only if $W - w > 1/(4\pi)$.

If $q < 1/2$, h agents are scarce, so that $w(h) = 2w$ and $w(\ell) = 0$. As $e^{TU} = \min\{2w; 1\}$, $q = \pi e^{TU} < 1/2$ only if $\min\{2w; 1\} < 1/(2\pi)$.

Finally, if $q = 1/2$ a continuum of wages is consistent with a stable allocation: $w(h) \in [W, 2w]$ and $w(\ell) = 2w - w(h)$. Agents choose $e^{TU} = 2(w(h) - w)$. $q = \pi 2(w(h) - w)$ is equal to 1/2 only if $w(h) = w + 1((4\pi)$. Therefore $e^{TU} = 1/(2\pi)$ in this case.

Suppose first $q = \pi 2(w(h) - w) > 1/2$. This is only consistent with $\pi 2(W - w) > 1/2$. $q < 1/2$ is only consistent with $\pi \min\{2w; 1\} < 1/2$. For intermediate cases, that is $\pi(W - w) < 1/4 < \pi \min\{w; 1/2\}$, $q = 1/2$. Therefore $e = 2w(h) - 2w = 1/(2\pi)$, that is $w(h) = w + \pi/4$.

Proof of Corollary 1

When $W - w > 1/(4\pi)$, the property DD ($2w > W$) implies that $W > 1/(2\pi)$. In this case, $e^{LF} - e^{TU} = 2w - W > 0$.

When $\min\{w; 1/1\} < 1/(4\pi)$, $W < 1/(2\pi)$ since $2w > W$ and $W < 1$. In this case, $e^{LF} - e^{TU} = W - 2w < 0$.

In the intermediate case $W - w < 1/(4\pi) < \min\{w; 1/2\}$, $e^{TU} = 1/(2\pi)$ and therefore $e^{LF} - e^{TU}$ is positive only if $W > 1/(2\pi)$.

Proof of Proposition 1

Given an achievement based policy, which assigns h agents to ℓ agents whenever possible, an agent chooses effort e to solve

$$\begin{aligned} &\max_e \quad e\left(\frac{1-q}{q}w + \frac{2q-1}{q}W\right) + (1-e)w - \frac{e^2}{2} \text{ if } q > 1/2, \\ &\max_e \quad ew + (1-e)\frac{q}{1-q}w - \frac{e^2}{2} \text{ if } q \leq 1/2. \end{aligned} \tag{13}$$

Supposing $q > 1/2$, a necessary condition for investment is

$$e = \frac{2q-1}{q}(W - w).$$

In equilibrium $\pi e = q$ must hold. Since e above increases in q and πe increases in π, it is sufficient to verify that $q > 1/2$ can occur when $\pi = 1$. $e = q$ implies

$$q^2 - 2(W - w)q + (W - w) = 0$$

but the discriminant is $(W - w)^2 - (W - w) = (W - w)(W - w - 1) < 0$ since $W < 1$. Therefore in any equilibrium $q \leq 1/2$ and

$$e^A = \frac{1-2q}{1-q}w < w. \tag{14}$$

Replacing e^A by q/π and solving for q yields the expression in the proposition (the other solution is greater than 1). Clearly, the solution is less than 1/2. Since $e^A < w$ both $e^A < e^{LF}$ and $e^A < e^{TU}$. Under an achievement based policy investments satisfy (14). With $q = \pi e^A$

$$e^A = w + \frac{1}{2\pi} - \sqrt{w^2 + \frac{1}{4\pi^2}}.$$

Simple calculations show that the derivative of e^A with respect to π is negative, and the derivative of $q = \pi e^A$ with respect to π is positive.

Proof of Corollary 2

The condition $S^A > S^{LF}$ holds, if e^A solves the quadratic equation ${e^A}^2 - 4we^A + W^2 < 0$. Solving yields

$$e^A > 2w - \sqrt{4w^2 - W^2}. \tag{15}$$

Since $e^A < w$, $W^2 \geq 3w^2$ implies $S^A < S^{LF}$, that is $W_0 < \sqrt{3}w$. Finally, using (1), $W_0(w, \pi)$ solves

$$W(2w - W) = (W - e^A)2w - \frac{1}{2}(W^2 - {e^A}^2) \tag{16}$$

By Proposition 1, differentiating q with respect to w, and using $q = \pi e^A$, e^A is an increasing function of w. Therefore the RHS of (16) decreases in w, and increases in W since $W < 1$. The LHS increases in w and decreases in W since $w < W$. Hence as w increases, so must W to restore equality. Hence, $W_0(w, \pi)$ increases in w. It decreases in π, since e^A decreases in π, hence the RHS of (16) increases in π, and W must decrease to restore equality.

Proof of Proposition 3

The case of laissez-faire has been established in the text.

Under an achievement based policy agents segregate in background when possible. By Proposition 1 less than half of the p agents attain h. The remaining (ℓ, p) agents match with (h, u) agents if possible, since $w - f > 0$. Therefore $e < e^A(\pi) \leq w$ for p agents, since (ℓ, p) agents' expected wages exceed $w\pi e^A/(1 - \pi e^A)$. $e \leq w - f$ for u agents, which holds with equality if (ℓ, p) outnumber (h, u) agents.

Turn now to a background based policy. Denote effort investments by e_p for p and by e_u for u agents. Let $\pi > 1/2$. Then a p agent has a chance of $(2\pi - 1)/\pi$ of matching with a p agent. Otherwise he matches with a u agent, and, if own achievement is h, obtains a h match with probability $\min\{e_u/e_p; 1\}$, and with probability $\max\{(e_u - e_p)/(1 - e_p); 0\}$ if own achievement is ℓ. Hence, a p agent's effort choice solves

$$\max_e e\left(\frac{2\pi - 1}{\pi}W + \frac{1-\pi}{\pi}\left(\min\left\{\frac{e_u}{e_p}; 1\right\}(W - w) + w - f\right)\right)$$
$$+(1-e)\frac{1-\pi}{\pi}\max\left\{\frac{e_u - e_p}{1 - e_p}; 0\right\}(w - f) - \frac{e^2}{2}.$$

A u agent solves

$$\max_e e\left(\min\left\{\frac{e_p}{e_u};1\right\}(W-w)+w-f\right)+(1-e)\max\left\{\frac{e_p-e_u}{1-e_u};0\right\}(w-f)-\frac{e^2}{2}.$$

Investment incentives of p and u agents can only be aligned if $\pi = 1/2$. Therefore $e_p \neq e_u$ for $\pi \neq 1/2$. Suppose $e_p > e_u$. Then

$$\begin{aligned} e_p &= W - \frac{1-\pi}{\pi}\left(\frac{e_p-e_u}{e_p}(W-w)+f\right) > w-f, \\ e_u &= W-w+\frac{1-e_p}{1-e_u}(w-f) > W-F. \end{aligned}$$

Suppose $e_p < e_u$. Then

$$\begin{aligned} e_p &= W - \frac{1-\pi}{\pi}\left(f+\frac{e_u-e_p}{1-e_p}(w-f)\right) > W-F, \\ e_u &= w-f+\frac{e_p}{e_u}(W-w) > w-f. \end{aligned}$$

Since education inputs are substitutes under property DD ($2w > W$), both a regime where p agents invest a lot and u agents a little, and the reverse may emerge in equilibrium, both are possible for π sufficiently close to 1/2.

Let now $\pi \leq 1/2$. Then a p agent matches with a u agent with certainty, and, if own achievement is h, obtains a h match with probability $\min\{e_u/e_p;1\}$, and with probability $\max\{(e_u-e_p)/(1-e_p);0\}$ if own achievement is ℓ. Hence, a p agent solves

$$\max_e e\left(\min\left\{\frac{e_u}{e_p};1\right\}(W-w)+w-f\right)+(1-e)\max\left\{\frac{e_u-e_p}{1-e_p};0\right\}(w-f)-\frac{e^2}{2}.$$

A u agent is assigned to a p agent with probability $\pi/(1-\pi)$ and otherwise matches with a u agent. Therefore a u agent solves

$$\begin{aligned} \max_e e\left(\frac{1-2\pi}{1-\pi}(W-F)+\frac{\pi}{1-\pi}\left(\min\left\{\frac{e_p}{e_u};1\right\}(W-w)+w-f\right)\right) \\ +(1-e)\frac{\pi}{1-\pi}\max\left\{\frac{e_p-e_u}{1-e_u};0\right\}(w-f)-\frac{e^2}{2}. \end{aligned}$$

First order conditions imply that $e_p \neq e_u$ for $\pi \neq 1/2$. Supposing $e_p > e_u$

$$\begin{aligned} e_p &= w-f+\frac{e_u}{e_p}(W-w) > w-f, \\ e_u &= W-\frac{1-2\pi}{1-\pi}F-\frac{\pi}{1-\pi}\left(w-\frac{1-e_p}{1-e_u}(w-f)\right) > W-F. \end{aligned}$$

Otherwise, if $e_p < e_u$

$$e_p = W - w + \frac{1-e_u}{1-e_p}(w-f) > W - F,$$
$$e_u = \frac{1-2\pi}{1-\pi}(W-F) + \frac{\pi}{1-\pi}\left(\frac{e_p}{e_u}(W-w) + w - f\right) > e_p > W - F.$$

Again both regimes are possible for π sufficiently close to 1/2.

Proof of Proposition 4

As established in the text integration is a stable labor market outcome with market wages $w(\ell) = 2w - w(h)$ and (i) $w(h) = W$ if $q > 1/2$, (ii) $w(h) \in [W, 2w]$ if $q = 1/2$, and (iii) $w(h) = 2w$ if $q < 1/2$. An investing agent solves

$$\max_e ew(h) + (1-e)w(\ell) - \frac{e^2}{2} - g(b, b').$$

Optimal interior investments satisfy

$$e = \begin{cases} e_0 = 2(W-w) & \text{if } q > 1/2 \\ [2(W-w), \min\{2w; 1\}] & \text{if } q = 1/2 \\ e_1 = \min\{2w; 1\} & \text{if } q < 1/2 \end{cases}$$

Strictly positive investment is profitable if $e^2/2 > g(b, b')$.

Denote by s_u the measure of agents in (u, u) schools. $q > 1/2$ implies $e = 2(W-w)$, so that (u, u) agents do not invest as $F > W^2/2$. This is only consistent if $(1-s_u)e > 1/2$, that is $(1-s_u)2(W-w) > 1/2$.

$q \leq 1/2$ implies $e = \min\{2w; 1\}$, so that (u, u) agents invest as $F < 2w^2$. This is only consistent if $\min\{2w; 1\} \leq 1/2$, that is $w \leq 1/4$.

If $1/4 \geq w \geq W - 1/(4(1-\rho))$, $q = 1/2$ must hold. To have $q = 1/2$ either $(1-s_u)2(w(h)-w) = 1/2$ if $1/(4(1-s_u)) \in [W-w, \sqrt{F}]$, or $2(w(h)-w)^2 = F$ and measure $1/2 - (1-s_u)\sqrt{F}$ of (u, u) agents invest $e = \sqrt{F}$, leaving them indifferent between $e > 0$ and $e = 0$.

s_u is determined at the school stage. If $q > 1/2$ payoffs at school are

$$2w - W \text{ if } g(b, b') = F,$$
$$2w - W + 2(W-w)^2 - g(b, b') \text{ if } g(b, b') < F,$$

Hence, a u agent values a p agent at $2(W-w)^2 - f$, and a p agent values a u agent at $-f$. Schools integrate, that is $s_u = \max\{1-2\pi; 0\}$, if

$$(W-w)^2 > f,$$

which is implied by Assumption 1.

Using this and $(1-s_u)2(W-w) \geq 1/2$ gives condition (i) in the statement.

If $q < 1/2$ $e = 2w$ for all agents as above and schools integrate as $2f < F$.

If $q = 1/2$, payoffs at school are $2w - w(h) + 2(w(h) - w)^2 - g(b, b')$ if $g(b, b') < F$ and otherwise $2w - w(h)$, or $F - g(b, b')$ for all agents. Since $2f < 2(W - w)^2 \leq 2(w(h) - w)^2 \leq F$ under Assumption 1, schools integrate and $s_u = \max\{1 - 2\pi; 0\}$. Therefore, if $\pi \geq 1/2$ all agents invest $e = 2(w(h) - w) = 1/2$. Let $\pi < 1/2$. If $1/(4\pi) \leq \sqrt{2F}$ only agents with $g(.) < F$ invest $e = 1/(4\pi)$, otherwise $e = \sqrt{2F}$ and also measure $1/2 - 2\pi\sqrt{2F}$ of (u, u) agents invest. This completes the argument for part (ii).

Proof of Proposition 5

Under a background based policy p agents solve

$$\max_e \begin{cases} ew - \frac{e^2}{2} & \text{if } \pi \leq 1/2 \\ e\left(\frac{1-\pi}{\pi}w + \frac{2\pi-1}{\pi}W\right) - \frac{e^2}{2} & \text{if } \pi > 1/2. \end{cases}$$

Interior solutions satisfy $e^B = w$ if $\pi \leq 1/2$, and $e^B = W - (1-\pi)(W-w)/\pi$ if $\pi > 1/2$. That is, $e^{LF} > e^B \geq w > e^A$ for $\pi \in (0, 1)$.

Derivation of the Cutoff $W_1(w, \pi)$

For $\pi < 1/2$ a background based dominates an achievement based policy. While both policies induce exactly the same matching – each P is matched with a U and there is the same measure of (h, ℓ) firms for a given e – since $e^B > e^A$, there are more integrated firms and, as $2w > W$, surplus is higher. Therefore $W_1(w, \pi) = w$ as claimed.

If $\pi > 1/2$ screening by background loses its effectiveness as a measure $2\pi - 1$ of P agents segregate in education outcome, unlike under an achievement based policy. Total surplus under a background based policy is

$$S^B = e^B \left((2\pi - 1)W + (1-\pi)2w - \pi\frac{e^B}{2} \right). \tag{17}$$

$S^B > S^A$ if and only if

$$\pi(e^B - e^A)\left(2w - \frac{1}{2}(e^B + e^A)\right) > (2\pi - 1)e^B(2w - W). \tag{18}$$

The LHS captures the gain through better incentive provision by a background based policy, while the RHS gives the benefit from rematching by an achievement based policy. Since (18) strictly relaxes as W increases,

$$S^B > S^A \Leftrightarrow W > W_1(w,\pi).$$

Straightforward calculation shows that $W_1(w,\pi)$ increases in π for $\pi \in [1/2,1]$. $W_1(w,1) = W_0(w,1)$, as for $\pi = 1$ a background based policy implies the laissez-faire outcome. Therefore $W_0(w,\pi) > W_1(w,\pi)$ for $\pi < 1$ and the difference decreases in π.

Finally, we show that $W \geq \sqrt{3}w$ implies $S^B > S^A$. Since e^B increases and e^A decreases in π, both output and incentive effect in condition (18) move in the same direction. Since W_1 increases in π, $W_1(w,\pi)$ may not be monotone in w. Solving the quadratic expression $S^B > S^A$ for e^A yields

$$e^A < 2w - \sqrt{4w^2 - \frac{2}{\pi}S^B}. \tag{19}$$

$2S^B > 3\pi w^2$ gives a sufficient condition:

$$\begin{aligned} & 2((2\pi-1)W + (1-\pi)w)((2\pi-1)W + (1-\pi)3w)/2 > 3\pi^2 w^2 \\ \Leftrightarrow\ & (2\pi-1)W^2 + 4(1-\pi)Ww > 3w^2. \end{aligned}$$

Solving this quadratic expression in W yields the condition

$$W > \left(\frac{\sqrt{3(2\pi-1) + 4(1-\pi)^2}}{2\pi-1} - 2\frac{1-\pi}{2\pi-1} \right) w.$$

Since $(\sqrt{3(2\pi-1)+4(1-\pi)^2} - 2(1-\pi))/(2\pi-1) < \sqrt{3}$ a sufficient condition for $S^B > S^A$ is $W \geq \sqrt{3}w$, that is $W_1(w,\pi) < \sqrt{3}w$.

Derivation of the Cutoff $W_2(w,\pi)$

Compare now a background based policy to laissez-faire when $\pi \leq 1/2$. Under laissez-faire there are $\pi e^{LF}/2$ firms of type (h,h) and total surplus is $S^{LF} = \pi W^2/2$. With a background based policy there are measure πe^B of (h,ℓ) firms and total surplus is $S^B = \pi 3w^2/2$. Therefore, when $\pi \leq 1/2$, $S^{LF} > S^B$ if and only if $W > \sqrt{3}w$; hence $W_2(w,\pi) = \sqrt{3}w$ as claimed.

Consider now the case $\pi > 1/2$. In this case, $S^{LF} > S^{BB}$ if and only if

$$(e^{LF} - e^B)\left[(1-\pi)2w + (2\pi-1)W - \frac{\pi}{2}\left(e^{LF} + e^B\right)\right] > (1-\pi)e^{LF}(2w - W). \tag{20}$$

Manipulating condition (20) and solving for W yields

$$W > \frac{4\pi - 2 + \sqrt{1 - 4\pi + 7\pi^2}}{3\pi - 1} w := W_2(w, \pi).$$

Clearly, $W_2(w, \pi)$ increases in w and simple calculation reveals that W_2 increases also in π. Note that $W_2(w, \pi) \to 2w$ as $\pi \to 1$. Bounds on W_2 for $\pi \in [1/2, 1]$ are given by

$$\sqrt{3}w = W_2(w, 1/2) \leq W_2(w, \pi) \leq W_2(w, 1) = 2w.$$

$W_2(w, \pi) \geq \sqrt{3}w$ implies that $W_2(w, \pi) > W_0(w, \pi)$ (see Corollary 2).

Proof of Proposition 6

By (3) given s an agent with fixed cost f invests if

$$e^{AI} > \sqrt{2f} \Leftrightarrow e^A(s) > \sqrt{2f}.$$

Since $e^A(s)$ strictly decreases in s, $e^A(\max\{2\pi - 1; 0\}) < \sqrt{2f}$ implies that (u, p) agents do not invest if $s = \max\{2\pi; 0\}$ which is consistent therewith. If $e^A(\min\{2\pi; 1\}) > \sqrt{2f}$ (u, p) agents invest at $s = \min\{2\pi; 1\}$. For intermediate f, s is defined by $e^A(s) = \sqrt{2f}$ making (u, p) agents indifferent between investing or not, which is consistent with $\max\{2\pi; 0\} < s < \min\{2\pi; 1\}$. Aggregate surplus under AI is

$$\max\{2\pi - 1; 0\} e^{AI} \left(2w - \frac{e^{AI}}{2}\right) \text{ if } e^A(\max\{2\pi - 1; 0\}) < \sqrt{2f},$$
$$\min\{2\pi; 1\} e^{AI} \left(2w - \frac{e^{AI}}{2}\right) - 2\min\{\pi; 1-\pi\} f \text{ if } e^A(\min\{2\pi; 1\}) > \sqrt{2f}.$$

$S^A > S^{AI}$ if and only if $q^A(2w - e^A(s^A)/2) > q^{AI}(2w - e^A(s^{AI})/2)$. For $e^A(\max\{2\pi - 1; 0\}) < \sqrt{2f}$, $q^A = \pi e^A(\pi) > q^{AI}$ and $e^A(\pi) < e^A(s^{AI})$, therefore $S^A > S^{AI}$. Let now $e^A(\min\{2\pi; 1\}) > \sqrt{2f}$ and suppose $\pi \leq 1/2$ first. Then a sufficient condition for $S^{AI} > S^A$ is

$$2w\left(2e^A(2\pi) - e^A(\pi)\right) > 2(e^A(2\pi))^2 - \frac{(e^A(\pi))^2}{2}$$
$$\Leftrightarrow \quad 4\pi w(q^{AI} - q^A) > (q^{AI})^2 - (q^A)^2,$$

which must be true since $e^A(2\pi) < e^A(\pi) < w$. In case $\pi > 1/2$ a sufficient condition for $S^{AI} > S^A$ is

$$2w\left(q^{AI} - q^A\right) > \left(\frac{3}{2} - \pi - \frac{1}{2\pi}\right)(q^A(1))^2 + \frac{(q^{AI})^2 - (q^A)^2}{2\pi}.$$

This is implied by

$$w\left(1-\frac{q^A}{q^{AI}}\right) > \left(3-2\pi-\frac{1}{\pi}\right)q^{AI}.$$

Since $w \geq e^{AI} = q^{AI}$ and for $1/2 < \pi \leq 1$

$$2\pi + \frac{1}{\pi} - 2 > 1 > \frac{q^A}{q^{AI}},$$

$S^{AI} > S^A$ follows.

Proof of Proposition 7

A background based policy exhausts all matches of u and p, using uniform rationing. Hence, a p agent gets a u match with probability $\min\{(1-\pi)/\pi; 1\}$, and a u agent gets a p match with probability $\min\{\pi/(1-\pi); 1\}$. As these probabilities do not depend on schools, p agents in (p,p) and (u,p) schools have the same optimization problem, and choose the same investment e_p. Let e_u denote investment of u agents in (u,p) schools; (u,u) pupils do not invest.

The following establishes that $e_u = e_p = w$ are optimal investments and unique for $1 > W + w$, which is assumed for the following. As in the proof of Proposition 3 independence of rationing can be exploited. Four cases arise. Suppose first $\pi > 1/2$. Then, if $e_u \leq e_p$,

$$e_p = W - \frac{1-\pi}{\pi}\left(1-\frac{e_u}{e_p}\right)(W-w) \text{ and } e_u = W - \frac{e_p - e_u}{1-e_u}w.$$

This defines the functions

$$e_p(e_u) = \frac{W + e_u^2 - (1+W-w)e_u}{w} \text{ and } e_u(e_p) = e_p - \frac{\pi}{1-\pi}\frac{e_p(W-e_p)}{W-w}.$$

$W = e_p(e_u(W))$. $e_u(e_p)$ strictly increases in e_p for $e_u(e_p) > 0$, while $e_p(e_u)$ strictly decreases in e_u for $1 > W + w$ and $e_u \leq W$. Since optimality requires $0 < e_p, e_u \leq W$ the fixed point at W is unique for $e_u \leq e_p$.

If $e_u \geq e_p$

$$e_p = W - \frac{1-\pi}{\pi}\frac{e_u - e_p}{1-e_p}w \text{ and } e_u = W - \left(1-\frac{e_p}{e_u}\right)(W-w).$$

This defines the functions

$$e_p(e_u) = \frac{e_u^2 - we_u}{W-w} \text{ and } e_u(e_p) = \frac{1-\pi}{\pi}\frac{W + e_p^2 - e_p\left(1+W-\frac{\pi}{1-\pi}w\right)}{w}.$$

As optimality requires $e_u \geq w$, $e_p \leq W$ and $1 > W+w$, $e_p(e_u)$ strictly increases in e_u, while $e_u(e_p)$ strictly decreases in e_p. Therefore $e_u = W = e_p$ is the unique fixed point in this case.

Let now $\pi \leq 1/2$. Then, if $e_u \leq e_p$,

$$e_p = W - \left(1 - \frac{e_u}{e_p}\right)(W - w) \text{ and } e_u = W - \frac{\pi}{1-\pi}\frac{e_p - e_u}{1 - e_u}w.$$

This defines the functions

$$e_u(e_p) = \frac{e_p^2 - we_p}{W - w} \text{ and } e_p(e_u) = \frac{\pi}{1-\pi}\frac{W + e_u^2 - e_u\left(1 + W - \frac{1-\pi}{\pi}w\right)}{w}.$$

Since $\pi < 1 - \pi$ an analogous argument as in the second case above ensures that under $1 > W - w$ $e_u = W = e_p$ is the unique fixed point in this case as well.

Finally, if $e_u \geq e_p$

$$e_p = W - \frac{e_u - e_p}{1 - e_p}w \text{ and } e_u = W - \frac{\pi}{1-\pi}\left(1 - \frac{e_p}{e_u}\right)(W - w).$$

This defines the functions

$$e_u(e_p) = \frac{W + e_p^2 - (1 + W - w)e_p}{w} \text{ and } e_p(e_u) = e_u - \frac{1-\pi}{\pi}\frac{e_p(W - e_p)}{W - w}.$$

Here an analogous argument to the first case above can be used to establish uniqueness of the fixed point $e_u = W = e_p$ under $1 > W - w$ in this case.

Since optimal investments and uniform rationing under the background based policy ensure that complete segregation in achievement among the (u, p) matches on the labor markets is feasible, the statements in the proposition follow.

Omitted Details for Proposition 8

In the text it has been shown that $s_u > 0$ and $s_p + s_u < 1$. If $\pi > 1/2$ then $s_p = s_u + 2\pi - 1$ and a (u, u) agent is matched to a (p, p) agent for sure, does not invest, and obtains $e_p w$. For a (u, p) agent, who may match to a (p, p) or a (u, p) agent, positive investment solves

$$\max_e eW - \frac{e^2}{2},$$

supposing that at least (p,p) agents invest. Since $f < W^2/2$ all (u,p) agents invest $e = W$. In this case a (p,p) agent solves

$$\max_e e\left(\frac{s_p - s_u}{s_p}W + \frac{s_u}{s_p}w\right) - \frac{e^2}{2},$$

and therefore

$$e_p = \frac{s_p - s_u}{s_p}(W - w) + w.$$

That is, (u,p) invest more than (p,p) agents. No agent has an incentive to change schools if

$$W^2 - 2f \geq e_p^2 \text{ and } W^2 - 2f \geq 2e_p w,$$

with at most one strict inequality. Since $e_p < 2w$ the second condition must bind, that is $W^2 - 2f = 2e_p w > e_p^2$. This determines measures s_u and s_p since $s_u = s_p + 1 - 2\pi$ by feasibility, so that

$$s_p = (2\pi - 1)2w\frac{W - w}{W^2 - 2f - 2w^2}.$$

Note that $W^2 - 2f > 2w\left[W - \frac{1-\pi}{\pi}(W - w)\right]$ implies $2\pi - 1 < s_p < \pi$ and thus $0 < s_u < 1 - \pi$. Hence, measure $s_u e_p > 0$ of (h,ℓ), measure $(1 - s_u - s_p)W + (s_p - s_u)e_p > 0$ of (h,h), and measure $(s_u + s_p)W - s_p e_p > 0$ of (ℓ,ℓ) firms form.

Briefly consider the case $\pi \leq 1/2$. Suppose that $s_p = \pi$ and $s_u = 1 - \pi$ implying payoffs $w^2/2$ for (p,p) and $w^2\pi/(1-\pi)$ for (u,u) agents. Let a pair of agents matches into a (u,p) school. Since (p,p) agents are scarce these agents also match on the labor market implying optimal investments e_u, e_p satisfy $e_u = w - e_p(2w - W)$ and vice versa. That is, $e = w/(1 + 2w - W) < w$. Hence, segregation can be supported as an equilibrium outcome if

$$w^2\left(\frac{\frac{3}{2} + 2w - W}{(1 + 2w - W)^2} - \max\left\{\frac{1}{2}; \frac{\pi}{1-\pi}\right\}\right) > f.$$

Let now $s_u < 1 - \pi$ and $s_p < \pi$. Then (u,p) agents segregate and invest W, (p,p) invest w, and (u,u) agents invest 0 and obtain payoff $w^2 s_p/s_u$, where $s_p/s_u = 1 - (1 - 2\pi)/s_u$. Hence, $s_u < 1 - \pi$ can hold in equilibrium if

$$W^2 - w^2 \geq 2f \text{ and } W^2 - 2w^2\left(1 - \frac{1 - 2\pi}{s_u}\right) \geq 2f.$$

Hence, whenever $W^2 - 2f \geq w^2$ there exist $s_u < 1 - \pi$ such that $W^2 - 2w^2\left(1 - \frac{1-2\pi}{s_u}\right) \geq 2f$, in particular $s_u = 1 - 2\pi$, that is full school integration, can be supported as an equilibrium outcome.

Proof of Proposition 9

From above we know $e_p = (W^2 - 2f)/2w$. Assume that

$$W^2 - 2f > 2w \left[W - \frac{1-\pi}{\pi}(W - w) \right]. \tag{21}$$

Then aggregate surplus under a club based policy can be written as

$$\begin{aligned} S^C &= 2s_u e_p w + (s_p - s_u) e_p W + (1 - s_p - s_u) \left(\frac{W^2}{2} - f \right) - s_p \frac{e_p^2}{2} \\ &= s_p \frac{e_p^2}{2} + s_u e_p w + 2(\pi - s_p) \left(\frac{W^2}{2} - f \right) = s_p \frac{e_p^2}{2} + (1 - s_p) \left(\frac{W^2}{2} - f \right). \end{aligned}$$

Note that $S^C = s_p e_p^2/2 + (1 - s_p) w e_p$. We continue to show that for each labor market policy there exists some $\pi^* > 1/2$ with the property stated in the proposition.

Comparing to laissez faire, $S^C > S^{LF}$ if

$$\left(1 - s_p \left(1 - \frac{e_p}{2w}\right)\right) \left(\frac{W^2 - 2f}{2}\right) > \pi \frac{W^2}{2}.$$

That is,

$$1 - \pi \frac{W^2}{W^2 - 2f} > s_p \left(1 - \frac{e_p}{2w}\right), \tag{22}$$

Since $s_p \leq \pi$ (with equality if (21)) holds with equality), this is implied by

$$\frac{1}{\pi} > 1 + \frac{W^2}{W^2 - 2f} - \frac{W^2 - 2f}{4w^2}.$$

The RHS is less than 2 if $W^2 < 6f$, so that there is $\pi^* > 1/2$ such that (21)) holds with equality and $S^C > S^{LF}$. Hence, for any $1/2 \leq \pi < \pi^*$ (21)) continues to hold and $s_p < \pi$, which implies $S^C > S^{LF}$ at π.

Concerning an achievement based policy, $S^C > S^A$ if and only if

$$\left(1 - s_p \left(1 - \frac{e_p}{2w}\right)\right) w e_p > \pi e_A(\pi) \left(2w - \frac{e^A(\pi)}{2}\right).$$

Using that $e_p \geq w$ a sufficient condition is

$$\left(1 - \frac{s_p}{2}\right) w^2 > \pi e_A(\pi) \left(2w - \frac{e^A(\pi)}{2}\right).$$

If (21) holds $s_p \geq \pi$, so that $S^C > S^A$ if

$$\frac{1}{\pi} > \frac{1}{2} + \frac{e_A(\pi)}{w}\left(2 - \frac{e^A(\pi)}{2w}\right).$$

As $e_A < w$ there is $\pi^* > 1/2$ such that this condition holds with equality for π^* and with strict inequality for $1/2 \leq \pi < \pi^*$. Since S^C decreases in s_p and increases in e_p, the above condition holds for all $\pi \geq 1/2$ so that (21) holds.

Comparing to a background based policy, $S^C > S^B$ if and only if

$$s_p \frac{e_p^2}{2} + (1 - s_p) w e_p > \pi \frac{(e^B)^2}{2} + (1 - \pi) w e^B. \tag{23}$$

If (21) holds with equality, $s_p = \pi$ and $e_p = e_B$, and the above inequality holds with equality. This implies, as the LHS of (23) increases in e_p and $e_p = (W^2 - 2f)/2w$ that if (21) holds with strict inequality sign the same is true for the above condition. Hence, $S^C > S^B$ whenever (21) holds with strict inequality, and $\pi^* = 1$.

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