Optimal Tariffs on Exhaustible Resources: The Case of Quantity Setting

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Abstract

Constructing a dynamic game model of trade of an exhaustible resource, this paper compares feedback Nash and Stackelberg equilibria. We consider two different leadership scenarios: leadership by the importing country, and leadership by the exporting country. We numerically establish that each Stackelberg equilibrium involves higher payoffs for both the leader and follower, but reduces welfare of the rest of the world. The world welfare is largest in the Stackelberg equilibrium in which the importing country is a leader and smallest in the Nash equilibrium.

Keywords: dynamic game, feedback Nash equilibrium, feedback Stackelberg equilibrium.

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1 Introduction

The world markets for gas and oils consist mainly of a small number of large sellers and buyers. For instance, the U.S. Energy Information Administration reports that the major energy exporters concentrate on the Middle East and Russia whereas the United States, Japan and China have a substantial share in the imports. These data suggest that bilateral monopoly roughly prevails in the oil market in which the both parties exercise market power. What are the implications of market power for welfare of importing and exporting countries, and the world?

There is a large literature that attempts to answer the above question by using a dynamic game formulation. Newbery (1976) and Kemp and Long (1980) are among the earliest contributions, showing that the optimal tariff is time inconsistent in an open-loop Stackelberg equilibrium.[1] In order to overcome this difficulty, Karp and Newbery (1991, 1992) consider a feedback (Markovian) model in which importing countries play a dynamic game with perfectly competitive exporters. Karp and Newbery (1991) compare two situations, in one of which the importing countries are the first movers in each period, while in the other, the competitive exporters choose their outputs before the importing countries set their tariff rates. They numerically demonstrate that being the first-mover can be disadvantageous. Focusing on the Nash equilibrium, Karp and Newbery (1992) make a welfare comparison between free trade and the Markov perfect Nash equilibrium.

While Karp and Newbery (1991, 1992) assume price-taking suppliers, Wirl (1994) computes a feedback Nash equilibrium when both the importing and exporting countries have market power. His novel result is that resource extraction is more conservative than the globally efficient level, but that the equilibrium converges to the efficient steady state.[2] His model has been ex-

[1] The time consistency issue is further studied by Karp (1984) who assumes that production cost depends on the resource stock. Newbery (1981) did not deal with optimal tariff issues, but pointed another type of time inconsistency, when a cartel is the open loop Stackelberg leader and a fringe of competitive producers acts as the followers.

[2] In the steady state, a positive resource stock remains in the ground, even though

tended in several ways. Chou and Long (2009), maintaining the assumption of Nash behavior, extend the model to accommodate many importers and compare welfare in free trade and the Nash equilibrium. Tahvonen (1996) and Rubio and Escriche (2001) turn attention to Stackelberg games. Both papers show that outcome of the Nash equilibrium is identical to that of the Stackelberg equilibrium where the exporting country leads.[3]

This paper is also in line with the above literature, but our model and purpose are quite different. First, we consider the case where the seller chooses quantity whereas all of the above papers assume price-setting behavior. Given the fact that recent price fluctuations of oil are caused by quantity control by the resource-rich countries, our quantity-setting formulation seems more plausible. Second, we compare welfare of each country and the world in the Nash equilibrium and the two Stackelberg equilibria where the leadership role is taken by the importer and the exporter, respectively. Third and most importantly, we derive feedback Stackelberg equilibria which are conceptually different from Tahvonen (1996) and Rubio and Escriche (2001). Roughly speaking, they assume that the leader moves first in each period, but does not necessarily try to improve upon its Nash equilibrium payoff stream. Such a solution may be called a stagewise Stackelberg equilibrium. In contrast, since we suppose that the leader determines a Markovian rule over the entire horizon of the game, a solution concept that may be called a hierarchical Stackelberg equilibrium.[4] With these differences, we establish that (i) each Stackelberg solution yields higher welfare for both players as compared to the Nash equilibrium, and that (ii) the world welfare is highest in the Stackelberg equilibrium where the importing country is the leader. These findings are in sharp contrast to the results of Tahvonen (1996) and Rubio and Escriche (2001) that where the exporting country's welfare under

extraction is costless. This is because a Pigouvian tax that corrects stock-pollution externalities chokes off the demand.

[3]While Wirl assumes costless extraction, Tahvonen postulates a quadratic extraction cost function, and the other two papers assume stock-dependent cost.

[4]This concept is discussed in Dockner et al. (2000) in details. Basar and Olsder (1995) and Mehlmann (1988) name our solution concept the global Stackelberg solution.

its leadership is the same as in the Nash equilibrium. They are also in sharp contrast to the price-setting model of Fujiwara and Long (2009), where the world welfare is highest in the Nash equilibrium.[5]

This paper is organized as follows. Section 2 describes the model. Section 3 derives the feedback Nash equilibrium. Sections 4 characterizes the feedback Stackelberg equilibrium in which the importing country is the leader. Section 5, on the other hand, turns to the feedback Stackelberg equilibrium in which the exporting country leads. Section 6 presents numerical results showing that Stackelberg equilibria are Pareto superior to the Nash equilibrium. Section 7 concludes.

2 The Model

This section presents our basic model. There are three countries labeled Home, Foreign, and ROW (the rest of the world). A Foreign monopolistic firm produces and exports a good denoted by y to Home and ROW exclusively.[6] In producing the good, the Foreign firm extracts an exhaustible resource.

Due to geological factors, it is commonly observed that marginal extraction cost increases as the remaining stock of resource decreases.[7] This feature has been taken into account by various authors. Our formulation of extraction cost is closest to Karp (1984).

Let $\overline{X}$ be the initial size of the deposit and $X(t)$ be the stock of resource that remains at time t, and define $S(t) = \overline{X} - X(t) \geq 0$. Then, marginal extraction cost is increasing in S. Letting $y(t)$ denote the extraction at time

[5] Fujiwara and Long (2009) assume that the exporting country chooses prices, as in the cited papers.

[6] The good is not consumed in Foreign, and the market of Home and ROW is assumed to be integrated and hence the Foreign firm does not supply to each country separately.

[7] In a recent exposition of the state of the oil market, Smith (2009, p. 147) pointed out that "*most of the oil in any given deposit will never be produced, and therefore does not count as proved reserves, because it would be too costly to effect complete recovery.*" This indicates that the "exhaustion" of a deposit should be interpreted as an "economic abandonment" of the deposit after the profitable part has been exploited.

t, the cost of extracting $y(t)$ is assumed to be

$$C = [c_A + cS(t)]\, y(t),$$

where $c_A \geq 0$ and $c > 0$. In what follows, we set $c_A = 0$ for simplicity. Our results are not qualitatively affected even if c_A is positive.

Denote by a the maximum price that consumers would be willing to pay for the first unit of resource consumed at any t, which is called the choke price. It is clear if marginal cost of extraction, $cS(t)$, is higher than the choke price, it is socially inefficient to extract the resource. Therefore, extraction must stop as soon as $S(t)$ reaches the critical level $\overline{S} = a/c$ (if $\overline{X}$ is sufficiently large, so that S can reach a/c before exhaustion). In what follows, we assume that $\overline{X}$ is large enough, so that the resource stock is abandoned before exhaustion.[8]

The utility function of the two importing countries is specified by[9]

$$\begin{aligned} u^H &= aq_1^H - \frac{\left(q_1^H\right)^2}{2b} + q_2^H \qquad (1) \\ u^{ROW} &= aq_1^{ROW} - \frac{\left(q_1^{ROW}\right)^2}{2(1-b)} + q_2^{ROW}, \quad a > 0, \end{aligned}$$

where u^i, $i = H, ROW$ is utility of Home and ROW, and q_1^i and q_2^i are consumption of the imported good and numeraire good, respectively. The parameter $b \in (0, 1)$ represents the share of the Home demand in the world demand if there is no tariff. Assuming that the Home government imposes a specific tariff on the import of Good 1 and that ROW observes laissez-faire, utility maximization under the budget constraint yields the demand functions

$$q_1^H = b(a - p - \tau), \quad q_1^{ROW} = (1 - b)(a - p), \qquad (2)$$

where p is the world price of Good 1 and τ is the tariff imposed by Home. Letting y be the total supply of the Foreign firm, the market-clearing condition

[8] Karp (1984) also focuses on this case.

[9] In what follows, the time argument t is suppressed unless any confusion arises.

is

$$b(a-p-\tau)+(1-b)(a-p)=a-p-b\tau=y,$$

from which the inverse demand function is defined by $p=a-y-b\tau$. Substituting this into (2) and (1), and considering that Home's welfare W consists of consumer surplus and tariff revenue, we obtain

$$\begin{aligned} W &= aq_1^H - \frac{\left(q_1^H\right)^2}{2b} - (p+\tau)q_1^H + \tau q_1^H \\ &= \frac{b[y+(1+b)\tau][y-(1-b)\tau]}{2} \\ &= \frac{b\left[y^2+2b\tau y-\left(1-b^2\right)\tau^2\right]}{2}. \end{aligned} \tag{3}$$

On the other hand, the Foreign firm's profit π is

$$\pi=(a-b\tau-cS-y)y. \tag{4}$$

Home and Foreign strategically choose a time profile of τ and y by taking into account the resource dynamics in an infinite time horizon. Thus, the present model takes the form of the following dynamic game:

$$\begin{aligned} &\max_{\tau} && \int_0^\infty e^{-rt} W dt \\ &\max_{y} && \int_0^\infty e^{-rt} \pi dt \\ &\text{subject to} && \dot{S}=y, \quad S(0)=S_0: \text{ given}, \quad \lim_{t\to\infty} S(t) \le \frac{a}{c}. \end{aligned}$$

where $r>0$ is a common rate of discount. The subsequent sections find the Nash and Stackelberg solutions under linear feedback (Markovian) strategies.

3 Feedback Nash Equilibrium

This section considers a feedback Nash equilibrium of the above game. For this purpose, let us define each player's Hamilton-Jacobi-Bellman (HJB) equation. By the assumption of simultaneous moves, Home does not observe the firm's output $y(t)$ when it makes the tariff decision $\tau(t)$, and the Foreign firm makes its output decision without knowing the tariff rate $\tau(t)$.

Assume the Home government thinks that the Foreign firm has the output strategy $y = \phi(S)$ while the Foreign firms thinks that the Home country has the tariff strategy $\tau = \psi(S)$. Then, the two HJB equations are

$$\begin{aligned} rV(S) &= \max_{\tau} \left\{ \frac{b\left\{[\phi(S)]^2 + 2b\tau\phi(S) - (1-b^2)\,\tau^2\right\}}{2} + V_S(S)\phi(S) \right\} \\ rV^*(S) &= \max_{y} \left\{[a - b\psi(S) - cS - y + V_S^*(S)]y\right\}, \end{aligned} \tag{5}$$

where $V(S)$ and $V^*(S)$ are the value function of Home and Foreign. The first-order conditions for maximizing the right-hand side of the HJB equations give

$$\begin{aligned} \tau &= \frac{b\phi(S)}{1-b^2} \\ 2y &= a - b\psi(S) - cS + V_S^*(S). \end{aligned}$$

In equilibrium, what each player thinks about the other's strategy is correct and thus we have

$$\begin{aligned} \psi(S) &= \frac{b\phi(S)}{1-b^2} \\ 2\phi(S) &= [a - b\psi(S) - cS + V_S^*(S)]. \end{aligned}$$

Substituting these into the Foreign HJB equation, we obtain

$$rV^*(S) = [\phi(S)]^2. \tag{6}$$

Solving the above system determining $\psi(S)$ and $\phi(S)$ for $\phi(S)$ yields

$$\phi(S) = \frac{(1-b^2)\,(a - cS + V_S^*)}{2-b^2}. \tag{7}$$

Substituting (7) into (5), we obtain the following first-order differential equation:

$$rV^*(S) = \left[\frac{(1-b^2)\,(a - cS + V_S^*)}{(2-b^2)}\right]^2. \tag{8}$$

Let us guess that the value function is quadratic in S:

$$V^*(S) = \frac{A^*}{2}S^2 + B^*S + C^*.$$

Then, equation (8) becomes

$$r\left(\frac{A^*}{2}S^2+B^*S+C^*\right)=\left\{\frac{(1-b^2)[(A^*-c)S+B^*+a]}{(2-b^2)}\right\}^2.$$

Equating the coefficients of the terms S^2, S, and the constant terms on both sides of the equation, we get

$$\frac{rA^*}{2} = \left[\frac{(1-b^2)(A^*-c)}{(2-b^2)}\right]^2 \tag{9}$$

$$rB^* = \frac{2(1-b^2)^2(A^*-c)(B^*+a)}{(2-b^2)^2} \tag{10}$$

$$rC^* = \left[\frac{(1-b^2)(B^*+a)}{2-b^2}\right]^2. \tag{11}$$

Solving (9) for A^*, we have

$$A^* = \frac{4c(1-b^2)^2+r(2-b^2)^2\pm(2-b^2)\sqrt{\Delta}}{4(1-b^2)^2}$$

$$\Delta \equiv 8cr(1-b^2)^2+r^2(2-b^2)^2>0.$$

Noting that (7) gives the following equilibrium outputs:

$$\phi(S)=\frac{(1-b^2)[(A^*-c)S+B^*+a]}{2-b^2}\equiv\alpha^*S+\beta^*, \tag{12}$$

the resource dynamics is given by

$$\dot{S}=\frac{(1-b^2)[(A^*-c)S+B^*+a]}{2-b^2}.$$

Thus, we have to impose $A^*-c<0$ to guarantee the asymptotic stability and hence we must choose

$$A^*=\frac{4c(1-b^2)^2+r(2-b^2)^2-(2-b^2)\sqrt{\Delta}}{4(1-b^2)^2}. \tag{13}$$

Substituting (13) into (10) and (11), the other coefficients are derived as

$$B^* = \frac{\left[r(2-b^2)-\sqrt{\Delta}\right]a}{r(2-b^2)+\sqrt{\Delta}} \tag{14}$$

$$C^* = r\left[\frac{2(1-b^2)a}{r(2-b^2)+\sqrt{\Delta}}\right]^2. \tag{15}$$

Accordingly, in the Markov perfect Nash equilibrium (hereafter, MPNE), the Foreign firm's output strategy takes a form

$$\phi(S) = \alpha_N^* S + \beta_N^* = \alpha_N^* \left(S - \frac{a}{c}\right) \quad \text{for all} \quad S \leq \frac{a}{c} \tag{16}$$

$$\text{where} \quad \alpha_N^* = \frac{(1-b^2)(A^* - c)}{2-b^2} = \frac{r(2-b^2) - \sqrt{\Delta}}{4(1-b^2)} < 0 \tag{17}$$

$$\beta_N^* = \frac{(1-b^2)(B^* + a)}{2-b^2} = -\frac{\left[r(2-b^2) - \sqrt{\Delta}\right] a}{4c(1-b^2)} = -\frac{a\alpha_N^*}{c} > 0, \tag{18}$$

where the subscript N denotes the Nash equilibrium.

Eqs. (6) and (16) allow us to write

$$rV^*(S) = \left[\alpha_N^* \left(S - \frac{a}{c}\right)\right]^2 \text{ for all } S \leq \frac{a}{c} \tag{19}$$

When $S_0 = 0$, the Foreign firm's maximized profit becomes

$$V^*(0) = C^* = \frac{1}{r}\left(\frac{a\alpha_N^*}{c}\right)^2, \tag{20}$$

from (19).

It turns out to be more convenient to rewrite α_N^* as follows.

$$\alpha_N^* = \frac{1}{2}\left\{ r\left[\frac{2-b^2}{2(1-b^2)}\right] - \sqrt{r^2\left[\frac{2-b^2}{2(1-b^2)}\right]^2 + 2rc} \right\}$$

Based on these results, we can establish:

Proposition 1: *There exists a unique MPNE in linear strategies such that*

(i) the resource stock is abandoned when the stock-dependent marginal extraction cost reaches the choke price,

(ii) output decreases as the remaining stock decreases, and falls smoothly to zero at a constant rate α_N^*.

(iii) the per unit tariff rate decreases as the stock declines, and falls smoothly to zero when the resources stock is abandoned.

Proof: Part (i) has already been proved. Part (ii) follows by noting that

$$\dot{y} = \alpha_N^* \dot{S} = \alpha_N^* y.$$

Once this is confirmed, part (iii) is obvious because Home's optimal tariff is given by $\tau = by/\left(1 - b^2\right)$. **Q.E.D.**

The effect of the parameter b (market demand share of Home under no tariff) is summarized as follows.

Proposition 2: *α_N^* is increasing in b (i.e., the absolute value of α_N^* is decreasing in b.)*

Proof: If we define the function

$$\begin{aligned} g(\lambda) &= r\lambda - \sqrt{r^2\lambda^2 + 2rc} < 0 \\ \text{where} \quad \lambda &\equiv \frac{2 - b^2}{2(1 - b^2)} > 0, \end{aligned}$$

$d\alpha_N^*/db$ is expressed by $g_\lambda \cdot d\lambda/db$. Since straightforward calculations yield

$$\begin{aligned} g_\lambda &= r\left[1 - \frac{r\lambda}{\left(r^2\lambda^2 + 2rc\right)^{1/2}}\right] > 0 \\ \frac{d\lambda}{db} &= \frac{b}{(1 - b^2)^2} > 0, \end{aligned}$$

we can conclude that $d\alpha_N^*/db > 0$. **Q.E.D.**

Corollary: *The larger is b, the worse off is the resource exporting firm in the MPNE.*

Proof: This follows from Proposition 2 and Eq. (19). **Q.E.D.**

4 Feedback Stackelberg Equilibrium with Home's Leadership

This and the next sections turn to two Stackelberg equilibria. This section considers the case where the Home government is a leader. In order to solve the game backward, we begin by examining the Foreign firm's behavior. The Foreign firm anticipates that the leader chooses a strategy $\tau(S) = \alpha S + \beta$. Then, the Foreign firm's HJB equation is

$$rV^*(S) = \max_y \{[a - b(\alpha S + \beta) - cS - y]y + V_S^*(S)y\}.$$

Guessing $V^*(S) = A^*S^2/2 + B^*S + C^*$, the first-order condition for maximizing the right-hand side gives the follower's reaction function:

$$y(S) = \frac{(A^* - b\alpha - c)S + B^* + a - b\beta}{2}. \tag{21}$$

Substituting this into the HJB equation, we have

$$rV^*(S) = [y(S)]^2. \tag{22}$$

Applying this equation to the above specification of the value function, the three coefficients will be

$$\begin{aligned} A^* &= b\alpha + c + r - \sqrt{\Gamma} && (23) \\ B^* &= \frac{\left(r - \sqrt{\Gamma}\right)(a - b\beta)}{r + \sqrt{\Gamma}} && (24) \\ C^* &= \frac{1}{r}\left[\frac{\left(r - \sqrt{\Gamma}\right)(a - b\beta)}{2(b\alpha + c)}\right]^2 && (25) \\ \Gamma &\equiv r(2b\alpha + 2c + r) > 0. \end{aligned}$$

Substituting these into (21), the Foreign firm's strategy is

$$y(S) = \alpha^*S + \beta^* \equiv \frac{r - \sqrt{\Gamma}}{2}S - \frac{\left(r - \sqrt{\Gamma}\right)(a - b\beta)}{2(b\alpha + c)}. \tag{26}$$

Let us turn to the resolution of the leader's problem, which involves a few auxiliary steps. First, considering that the resource dynamics is expressed by $\dot{S} = \alpha^* S + \beta^*$, the solution is

$$S = e^{\alpha^* t}\left(S_0 + \frac{\beta^*}{\alpha^*}\right) - \frac{\beta^*}{\alpha^*}. \tag{27}$$

Second, under the linear strategies $\tau = \alpha S + \beta$ and $y = \alpha^* S + \beta^*$, the Home government's welfare flow at t with the resource stock S is

$$\begin{aligned} \frac{2W}{b} &= (\alpha^* S + \beta^*)^2 + 2b(\alpha S + \beta)(\alpha^* S + \beta^*) - \left(1 - b^2\right)(\alpha S + \beta)^2 \\ &= \left[\alpha^{*2} + 2b\alpha\alpha^* - \left(1 - b^2\right)\alpha^2\right] S^2 + 2\left[\alpha^*\beta^* + b(\alpha\beta^* + \alpha^*\beta) - \left(1 - b^2\right)\alpha\beta\right] S \\ &\quad + \beta^{*2} + 2b\beta\beta^* - \left(1 - b^2\right)\beta^2. \end{aligned}$$

Substituting (27) into this and making some rearrangements yield

$$\begin{aligned} \frac{2W}{b} &= \frac{-2(1-b^2)\alpha^2 + r(3b\alpha + c + r) - (2b\alpha + r)\sqrt{\Gamma}}{2} e^{\left(r - \sqrt{\Gamma}\right)t}\left(S_0 + \frac{\beta^*}{\alpha^*}\right)^2 \\ &\quad - \frac{\left[2(1-b^2)\alpha - b\left(r - \sqrt{\Gamma}\right)\right](\alpha a + \beta c)}{b\alpha + c} e^{\frac{r - \sqrt{\Gamma}}{2} t}\left(S_0 + \frac{\beta^*}{\alpha^*}\right) \\ &\quad - \left(1 - b^2\right)\left(\frac{\alpha a + \beta c}{b\alpha + c}\right)^2. \end{aligned}$$

Taking the integral of the discounted flow of welfare, we have

$$\begin{aligned} \int_0^\infty e^{-rt}\frac{2W}{b} dt &= \frac{-2(1-b^2)\alpha^2 + r(3b\alpha + c + r) - (2b\alpha + r)\sqrt{\Gamma}}{2\sqrt{\Gamma}}\left(S_0 + \frac{\beta^*}{\alpha^*}\right)^2 \\ &\quad - \frac{2\left[2(1-b^2)\alpha - b\left(r - \sqrt{\Gamma}\right)\right](\alpha a + \beta c)}{\left(r + \sqrt{\Gamma}\right)(b\alpha + c)}\left(S_0 + \frac{\beta^*}{\alpha^*}\right) \\ &\quad - \frac{1 - b^2}{r}\left(\frac{\alpha a + \beta c}{b\alpha + c}\right)^2. \end{aligned} \tag{28}$$

The Home government chooses α and β to maximize (28). Since this is virtually a static maximization problem, the optimal value of α and β is in principle obtained with calculus. However, one can see that the solutions of $\alpha$and β obtained through this method would depend on S_0, which implies that their choice can be time-inconsistent. In order to overcome this difficulty, we impose a time-consistency requirement: the restriction that $\alpha a + \beta c = 0$, so that the second term and the third terms in (28) vanish, and the first order condition becomes independent of S_o.

Under this restriction, the Foreign output is, from, (26),

$$y(S) = \alpha^* S + \beta^* \equiv \frac{r - \sqrt{\Gamma}}{2}(S - \frac{a}{c}), \tag{29}$$

and Foreign welfare is, using (22),

$$V^*(S) = \frac{1}{r}\left[\frac{r - \sqrt{\Gamma}}{2}(S - \frac{a}{c})\right]^2. \tag{30}$$

Comparison of (19) and (30) immediately yields:

Proposition 3: *Foreign welfare under Home leadership is higher than under MPNE iff*

$$(\alpha_N^*)^2 < \left(\frac{r - \sqrt{\Gamma}}{2}\right)^2. \tag{31}$$

With the time consistency condition, our maximization problem amounts to

$$\max_{\alpha} \quad \frac{-2(1-b^2)\alpha^2 + r(3b\alpha + c + r) - (2b\alpha + r)\sqrt{\Gamma}}{2\sqrt{\Gamma}} \left(S_0 - \frac{a}{c}\right)^2.$$

The first-order condition for this maximization problem is

$$2b(2b\alpha{+}2c{+}r)\sqrt{r(2b\alpha + 2c + r)} = -2\left(1 - b^2\right)\alpha(3b\alpha{+}4c{+}2r){+}rb(3b\alpha{+}5c{+}2r).$$

The rest of our task is to find α which satisfies this equation. Once it is found, β is obtained as $\beta = -\alpha a/c$. However, since it is not possible to obtain a closed form expression for α and β, we will proceed as follows. First, we prove the existence of a unique equilibrium and then proceed to numerical methods.

To show the existence and uniqueness of $\alpha > 0$ that solves the above first-order condition, it is convenient to define

$$\theta = 2b\alpha + 2c + r,$$

which is alternatively expressed as

$$\alpha = \frac{\theta - 2c - r}{2b}.$$

Then, the above first-order condition becomes

$$\begin{aligned}\frac{4r^{\frac{1}{2}}b^2\theta^{\frac{3}{2}}}{1-b^2} &= -3\theta^2 + \theta\left(\frac{3rb^2}{1-b^2} + 4c + 2r\right) + \left[\frac{rb^2}{1-b^2}(4c+r) + (2c+r)^2\right] \\ &\equiv -3\theta^2 + \eta\theta + \mu,\end{aligned}$$

where $\eta > 0$ and $\mu > 0$. The right-hand side of this equation is strictly concave in θ and its graph cuts the horizontal axis at a positive value, say, $\theta_H \equiv \left(-\eta + \sqrt{\eta^2 + 12\mu}\right)/6 > 0$, starting at a positive vertical intercept μ. On the other hand, the graph of the left-hand side is a strictly convex curve, starting at the origin. Therefore, we can find a unique value $\theta^* \in (0, \theta_H)$ at which both sides of the above equation are equal. This finding is summarized in:

Proposition 4. *There exists a unique feedback Stackelberg equilibrium in which Home (importing country) is a leader.*

5 Feedback Stackelberg Equilibrium with Foreign's Leadership

Finally, this section deals with the case in which the Foreign firm is a leader. Supposing that the leader's strategy is $y(S) = \alpha^* S + \beta^*$, the Home government's HJB equation is

$$rV(S) = \max_{\tau}\left\{\frac{b\left[(\alpha^*S + \beta^*)^2 + 2b\tau(\alpha^*S + \beta^*) - (1-b^2)\tau^2\right]}{2} + V_S(S)\left(\alpha^*S + \beta^*\right)\right\}.$$

The first-order condition for maximizing the right-hand side yields

$$\tau(S) = \frac{b(\alpha^*S + \beta^*)}{1-b^2}. \tag{32}$$

Substituting this into the definition of the Foreign firm's profit, we have

$$\pi = \left[a - \frac{b^2(\alpha^*S + \beta^*)}{1-b^2} - cS - \alpha^*S - \beta^*\right]\left(\alpha^*S + \beta^*\right),$$

which is rearranged to

$$\begin{aligned}\left(1-b^2\right)\pi &= -\alpha^*\left[\alpha^*+\left(1-b^2\right)c\right]S^2+\left[-2\alpha^*\beta^*+\left(1-b^2\right)(\alpha^* a-\beta^* c)\right]S \\ &\quad -\beta^*\left[\beta^*-\left(1-b^2\right)a\right].\end{aligned}$$

Noting that S depends on α^* and β^* in such a way that

$$S=e^{\alpha^* t}\left(S_0+\frac{\beta^*}{\alpha^*}\right)-\frac{\beta^*}{\alpha^*},$$

the above profit is rewritten further:

$$\left(1-b^2\right)\pi=-\alpha^*\left[\alpha^*+\left(1-b^2\right)c\right]e^{2\alpha^* t}\left(S_0+\frac{\beta^*}{\alpha^*}\right)^2+\left(1-b^2\right)(\alpha^* a+\beta^* c)\,e^{\alpha^* t}\left(S_0+\frac{\beta^*}{\alpha^*}\right).$$

Taking the integral from 0 to ∞, the Foreign firm's objective function becomes

$$\begin{aligned}\int_0^\infty e^{-rt}\left(1-b^2\right)\pi dt &= \frac{-\alpha^*\left[\alpha^*+\left(1-b^2\right)c\right]}{r-2\alpha^*}\left(S_0+\frac{\beta^*}{\alpha^*}\right)^2 \\ &\quad +\frac{\left(1-b^2\right)(\alpha^* a+\beta^* c)}{r-\alpha^*}\left(S_0+\frac{\beta^*}{\alpha^*}\right).\end{aligned}$$

In principle, we can find the equilibrium strategy of the leader by finding α^* and β^* which maximize this function. However, such solutions can be time-inconsistent for the same reason as in the preceding section. Therefore, we must impose a similar time consistency condition, $\alpha^* a+\beta^* c=0$. Under this condition, the problem at hand reduces to

$$V_L^*(S_0)\equiv\max_{\alpha^*}\left\{\frac{-\alpha^*\left[\alpha^*+\left(1-b^2\right)c\right]}{(r-2\alpha^*)(1-b^2)}\left(S_0-\frac{a}{c}\right)^2\right\}.$$

The first-order condition is

$$\frac{2\alpha^{*2}-2r\alpha^*-r(1-b^2)c}{(r-2\alpha^*)^2}=0,$$

which yields the solution:

$$\begin{aligned}\alpha_L^* &= \frac{r-\sqrt{\Phi}}{2}<0 \qquad (33)\\ \Phi &\equiv 2rc\left(1-b^2\right)+r^2>0,\end{aligned}$$

and Foreign's welfare under $S_0 = 0$:

$$V_L^*(0) = \frac{-\alpha_L^* \left[\alpha_L^* + (1-b^2)\, c\right]}{(r - 2\alpha_L^*)(1-b^2)} \left(\frac{a}{c}\right)^2.$$

Thus, we have arrived at:

Proposition 5. *Thee exists a unique feedback Stackelberg equilibrium in which Foreign (exporting country) is a leader.*

6 Numerical Examples and Interpretations

While the previous sections have focused on analytical derivations of the feedback solutions, it is generally quite difficult to characterize and compare them analytically. Therefore, this section makes use of a few numerical examples to obtain some insights on comparison among Nash and the two Stackelberg equilibria.[10] Our choice of parameters is as follows and our finding is summarized in Tables 1-6.

Example 1. $S_0 = 0, r = 0.1, c = 0.3$ and $b^2 = 0.5$,
Example 2. $S_0 = 0, r = 0.1, c = 0.3$ and $b^2 = 0.2$,
Example 3. $S_0 = 0, r = 0.1, c = 0.3$ and $b^2 = 0.01$.

These parameter values allow us to establish the following results. First, compared to the Nash equilibrium, two Stackelberg equilibria involve higher welfare of the follower as well as the leader while they reduce ROW's welfare. This implies that the presence of leadership in the resource markets is more desirable to both parties without any compensating transfer between them. Second, the sum of the three countries' welfare is largest in the Stackelberg equilibrium in which the importer is a leader. This finding is quite interesting since it suggests that the leadership of big buyers, e.g., the United States, Japan and China can be welfare-improving to the world.

[10] Detailed derivations are available from the authors upon request.

It is worth noting that these results crucially depend on the assumption that the exporting country chooses quantities. In a companion paper, Fujiwara and Long (2009) prove that the world welfare is lowest in the Stackelberg equilibrium where the importer leads and highest in the Nash equilibrium. Thus, we must know the exporter's strategy space (price or quantity decision rule) before drawing welfare implications.

7 Concluding Remarks

We have derived and explored feedback Stackelberg equilibria in a two-country dynamic game model of exhaustible resources. Unlike the existing literature that employs a stagewise Stackelberg solution, we have paid attention to the hierarchical Stackelberg equilibria.

Despite the above contributions, we have left much unexplored. In particular, we have restricted attention to linear strategies. However, Shimomura and Xie (2008) have provided an example of *renewable* resource exploitation in which there exist nonlinear feedback strategies that are superior to linear strategies.[11] Tackling this problem in the context of exhaustible resource markets is part of our future research agenda.

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ADDITIONAL NOTES

Whether the firm is at a MPNE or not, if it has an output strategy that is linear in S, i.e., $y = \alpha^* S + \beta^*$, its profit when the stock equals S is

$$\pi(S) = (p - cS)y(S) = [a - b\tau(S) - (\alpha^* S + \beta^*) - cS]\,(\alpha^* S + \beta^*)$$

and, if in addition $\tau(S) = \frac{by(S)}{1-b^2}$, then

$$\pi(S) = \left[a - \frac{b^2(\alpha^* S + \beta^*)}{1 - b^2} - cS - \alpha^* S - \beta^*\right](\alpha^* S + \beta^*)$$

So

$$\begin{aligned}\left(1 - b^2\right)\pi \quad &= \quad -\alpha^* \left[\alpha^* + \left(1 - b^2\right)c\right] S^2 + \left[-2\alpha^*\beta^* + \left(1 - b^2\right)(\alpha^* a - \beta^* c)\right] S \\ &\quad -\beta^* \left[\beta^* - \left(1 - b^2\right)a\right]\end{aligned}$$

Noting that S depends on α^* and β^* in such a way that

$$S = e^{\alpha^* t}\left(S_0 + \frac{\beta^*}{\alpha^*}\right) - \frac{\beta^*}{\alpha^*},$$

The above profit is rewritten further:

$$\left(1 - b^2\right)\pi = -\alpha^* \left[\alpha^* + \left(1 - b^2\right)c\right] e^{2\alpha^* t}\left(S_0 + \frac{\beta^*}{\alpha^*}\right)^2 + \left(1 - b^2\right)(\alpha^* a + \beta^* c)\, e^{\alpha^* t}\left(S_0 + \frac{\beta^*}{\alpha^*}\right).$$

Taking the integral from 0 to ∞, the Foreign firm's payoff becomes

$$\begin{aligned}(1 - b^2)V^*(S_0) \quad &= \quad \int_0^\infty e^{-rt}\left(1 - b^2\right)\pi dt = \frac{-\alpha^* \left[\alpha^* + (1 - b^2)\,c\right]}{r - 2\alpha^*}\left(S_0 + \frac{\beta^*}{\alpha^*}\right)^2 \\ &\quad + \frac{(1 - b^2)\,(\alpha^* a + \beta^* c)}{r - \alpha^*}\left(S_0 + \frac{\beta^*}{\alpha^*}\right).\end{aligned}$$

THIS IS TRUE WHETHER WE ARE AT A MPNE OR A STACKELBERG EQUILIBRIUM (except that at the MPNE, we must substitute α_N^* and β_N^* for α^* and β^*.

So let us substitute α_N^* and β_N^* for α^* and β^* into the above expression as a check, and take $S_0 = 0$ for simplicity.

$$J_X^{Nash}(0) = \frac{-\alpha_N^* \left[\alpha_N^* + (1-b^2)\, c\right]}{(r-2\alpha_N^*)(1-b^2)} \left(\frac{\beta_N^*}{\alpha_N^*}\right)^2 = \frac{-\alpha_N^* \left[\alpha_N^* + (1-b^2)\, c\right]}{(r-2\alpha_N^*)(1-b^2)} \left(\frac{a}{c}\right)^2$$

We must check to see if $C^* = J_X^{Nash}(0)$, i.e. verify that the following equation holds:

$$\frac{1}{r}\left(\frac{a\alpha_N^*}{c}\right)^2 - \left[\frac{-\alpha_N^* \left[\alpha_N^* + (1-b^2)\, c\right]}{(r-2\alpha_N^*)(1-b^2)} \left(\frac{a}{c}\right)^2\right] = 0 \tag{34}$$

where

$$\alpha_N^* = \frac{r(2-b^2) - \sqrt{\Delta}}{4(1-b^2)}$$

and

$$\Delta \equiv 8cr(1-b^2)^2 + r^2(2-b^2)^2$$

Equation (34) is equivalent to

$$(r-2\alpha_N^*)(1-b^2)\left(\alpha_N^*\right)^2 = -r\left(\alpha_N^*\right)^2 - r\left(1-b^2\right) c\alpha_N^*$$

i.e.

$$(r-2\alpha_N^*)(1-b^2)\alpha_N^* + r\alpha_N^* + r\left(1-b^2\right) c = 0$$

This quadratic equation in α_N^* is indeed satisfied by the value given by (**??**).

It also follows that when the Foreign firm is the leader, i.e., when it can choose α^* to maximize its payoff, it cannot do worse than the MPNE.

	α	α^*
Nash	-0.097034942	-0.068614066
Stackelberg (Home is leader)	-0.076613605	-0.071619488
Stackelberg (Foreign is leader)	-0.070710678	-0.05

Table 1: Example 1: α and α^* under $S_0 = 0, b^2 = 0.5, r = 0.1$ and $c = 0.3$

	α	α^*
Nash	-0.043896315	-0.078524116
Stackelberg (Home is leader)	-0.037126554	-0.079111676
Stackelberg (Foreign is leader)	-0.039363709	-0.070415945

Table 2: Example 2: α and α^* under $S_0 = 0, b^2 = 0.2, r = 0.1$ and $c = 0.3$

	α	α^*
Nash	-0.008296029	-0.082130692
Stackelberg (Home is leader)	-0.007207235	-0.082151291
Stackelberg (Foreign is leader)	-0.008254484	-0.081719398

Table 3: Example 3: α and α^* under $S_0 = 0, b^2 = 0.01, r = 0.1$ and $c = 0.3$

	Home	Foreign	ROW	Total
Nash	$0.155920291a^2$	$0.523098897a^2$	$0.129168596a^2$	$0.808187784a^2$
Home is leader	$0.160765394a^2$	$0.569927903a^2$	$0.105857077a^2$	$0.836550374a^2$
Foreign is leader	$0.160375076a^2$	$0.55555555a^2$	$0.081359222a^2$	$0.797289848a^2$

Table 4: Example 1: payoffs under $S_0 = 0, b^2 = 0.5, r = 0.1$ and $c = 0.3$

	Home	Foreign	ROW	Total
Nash	$0.074497901a^2$	$0.685115111a^2$	$0.115105577a^2$	$0.874718589a^2$
Home is leader	$0.074884995a^2$	$0.695406372a^2$	$0.108955935a^2$	$0.879247302a^2$
Foreign is leader	$0.074842868a^2$	$0.688667567a^2$	$0.098794532a^2$	$0.862304967a^2$

Table 5: Example 2: payoffs under $S_0 = 0, b^2 = 0.2, r = 0.1$ and $c = 0.3$

	Home	Foreign	ROW	Total
Nash	$0.014324174a^2$	$0.749494515a^2$	$0.130219746a^2$	$0.894038435a^2$
Home is leader	$0.014326649a^2$	$0.749870515a^2$	$0.129922493a^2$	$0.894119657a^2$
Foreign is leader	$0.014324359a^2$	$0.74950172a^2$	$0.129321342a^2$	$0.893147421a^2$

Table 6: Example 3: payoffs under $S_0 = 0, b^2 = 0.01, r = 0.1$ and $c = 0.3$